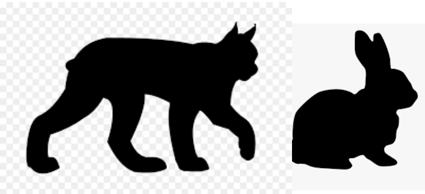
Compartmental models, Part II

Agenda:

1. Predator-prey models
2. SIR models
3. Modify the SIR to an SEIR model
4. Euler's method: how do we simulate the dynamics of an SIR model?

R tutorial



3. Two-population model
3. modèles de deux populations

The predator-prey model

<u>Compartmental models (Mechanistic Models)</u>

1. Populations are divided into compartments
2. Individuals within a compartment are homogenously mixed
3. Compartments and transition rates are determined by biological systems
4. Rates of transferring between compartments are expressed mathematically

How does the population of fossa regulate the population of lemurs in Ranomafana?

Comment la population de “fossa” régule la population de lemuriens à Ranomafana?

The predator-prey model

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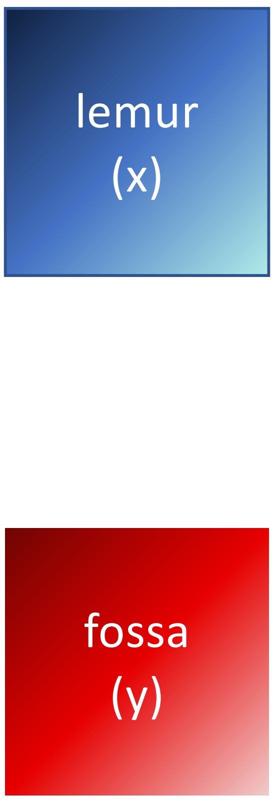

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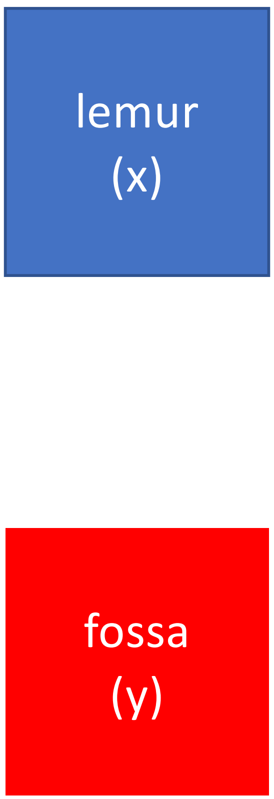

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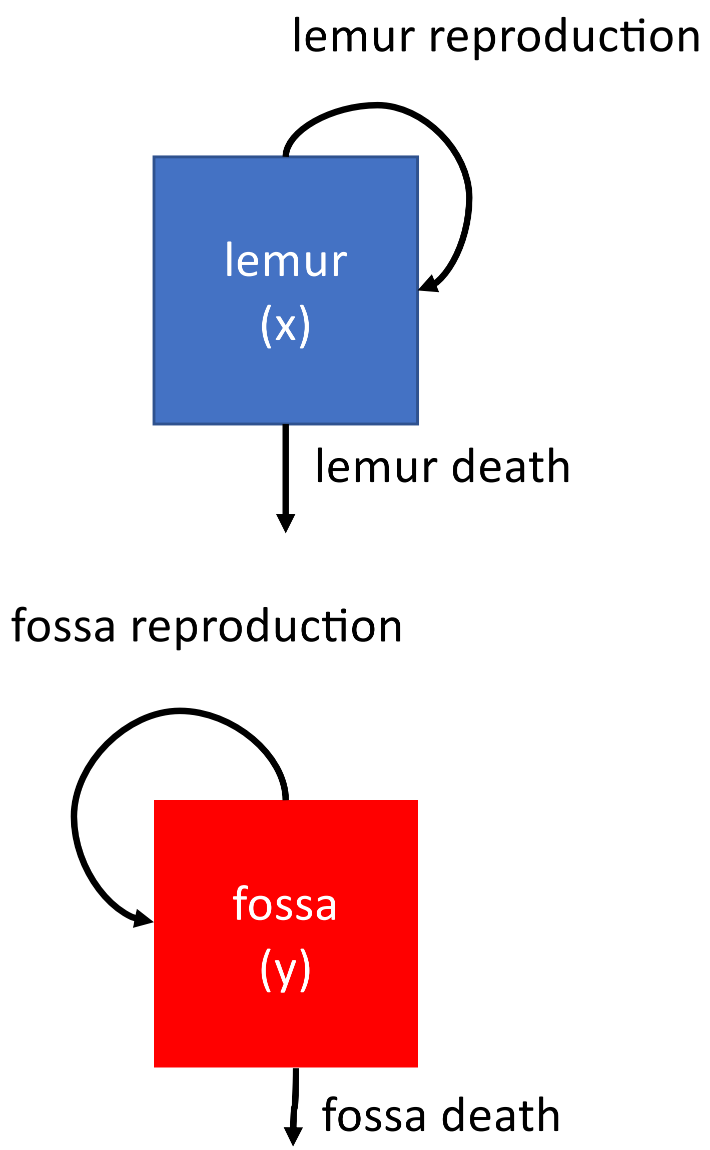

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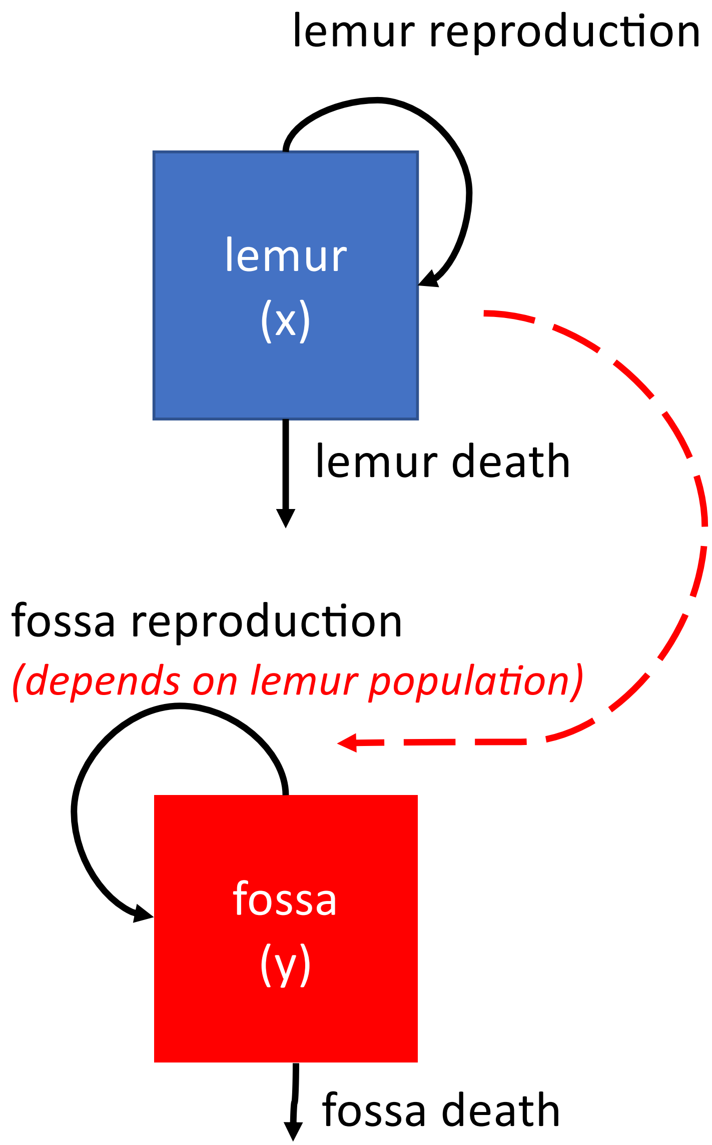

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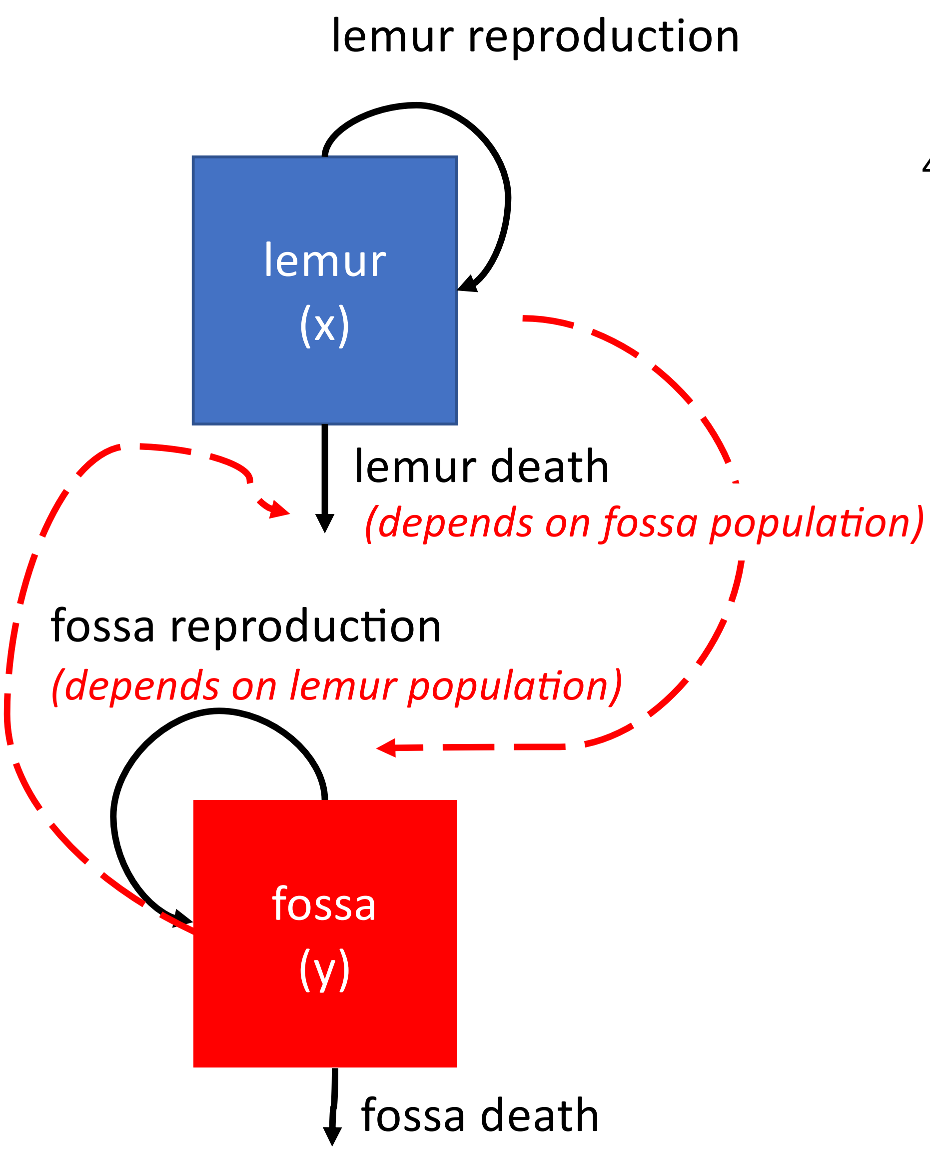

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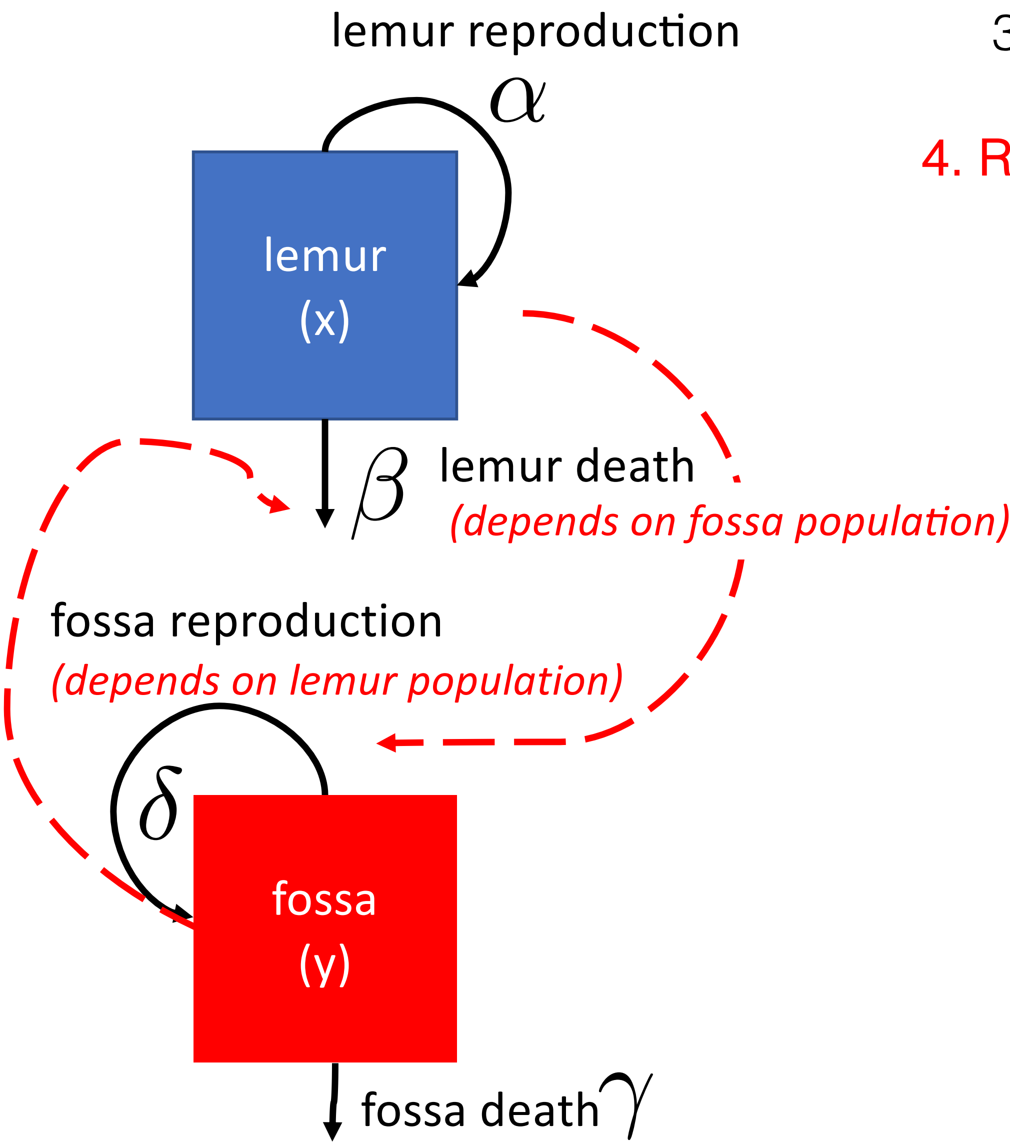

State variables	**Parameters**
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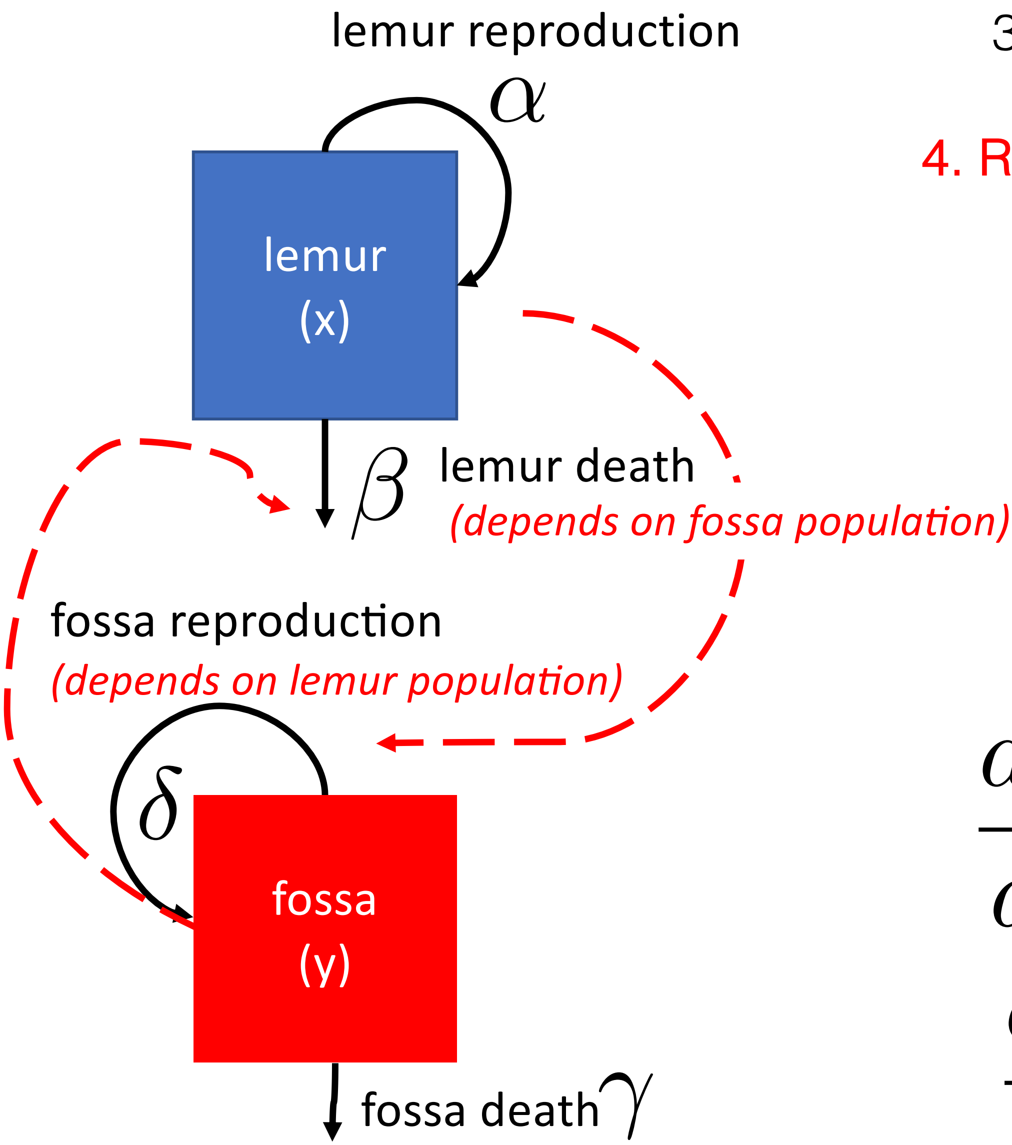
The predator-prey model



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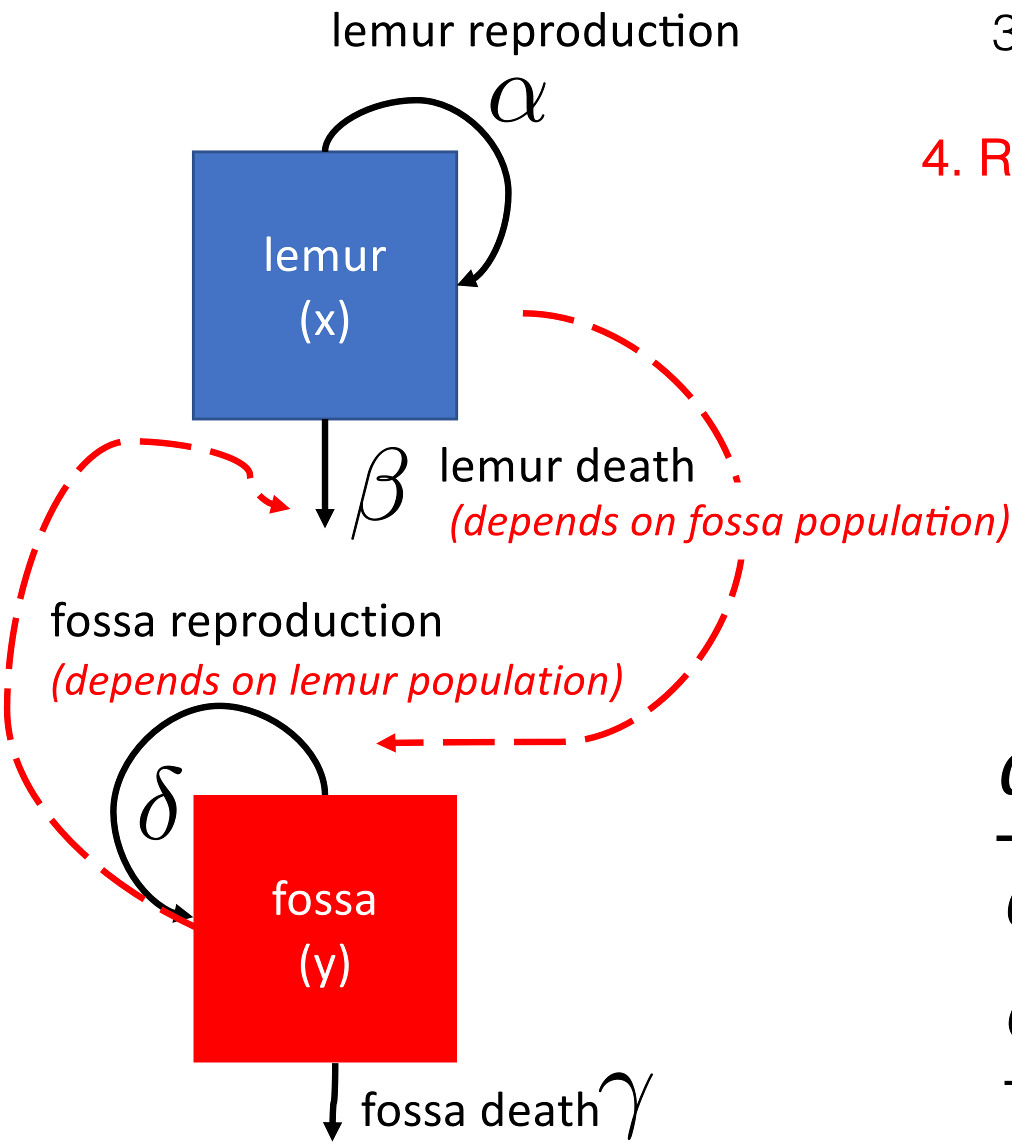

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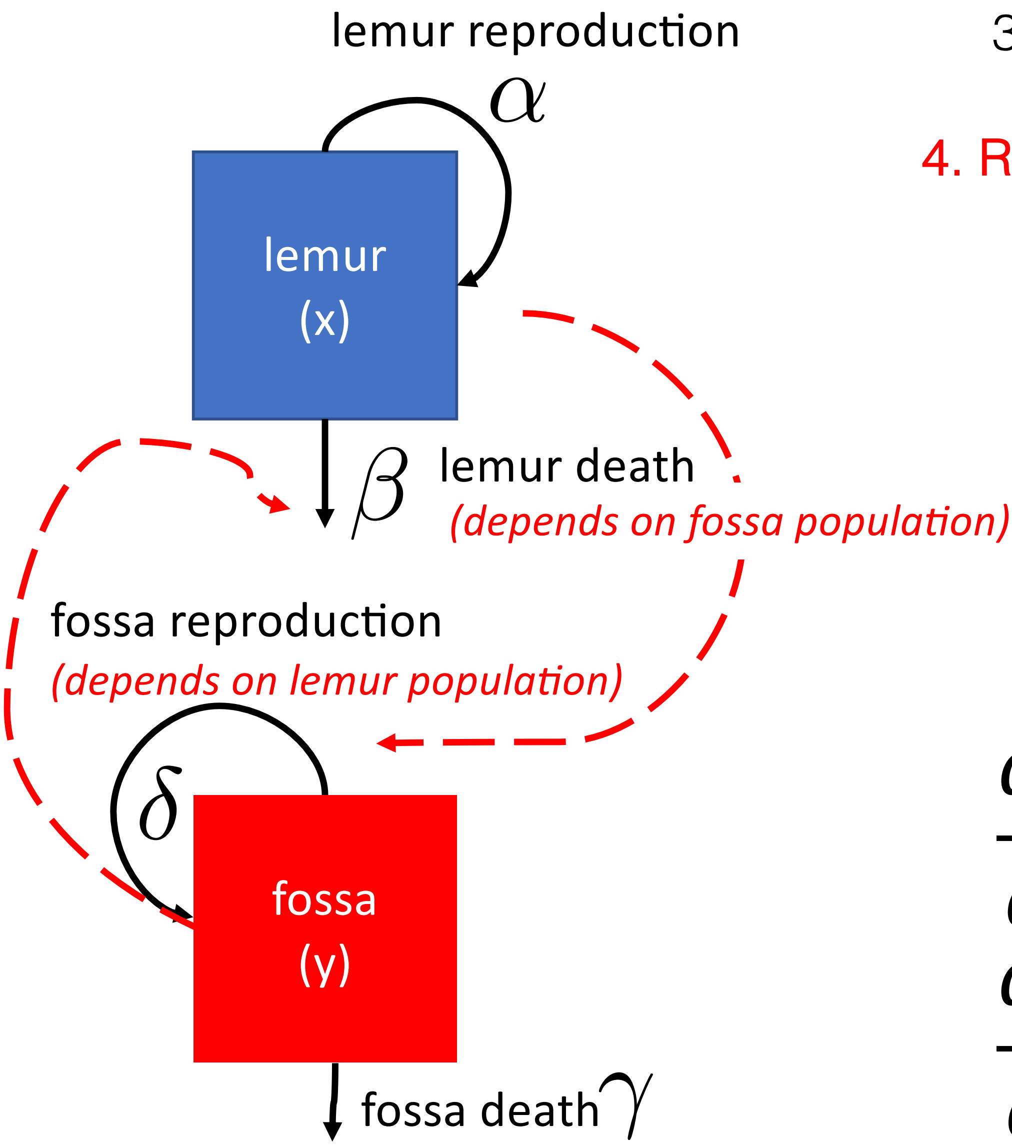

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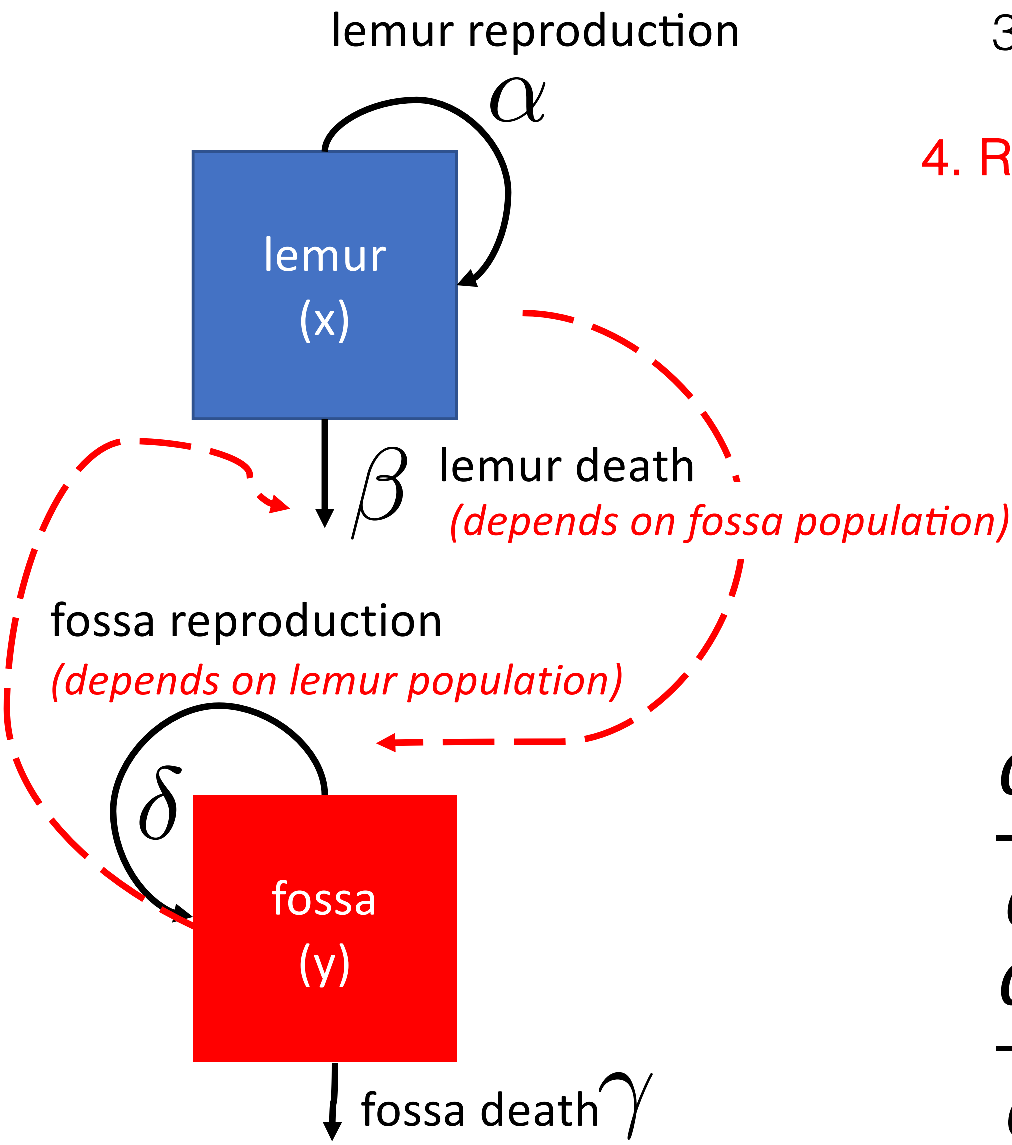

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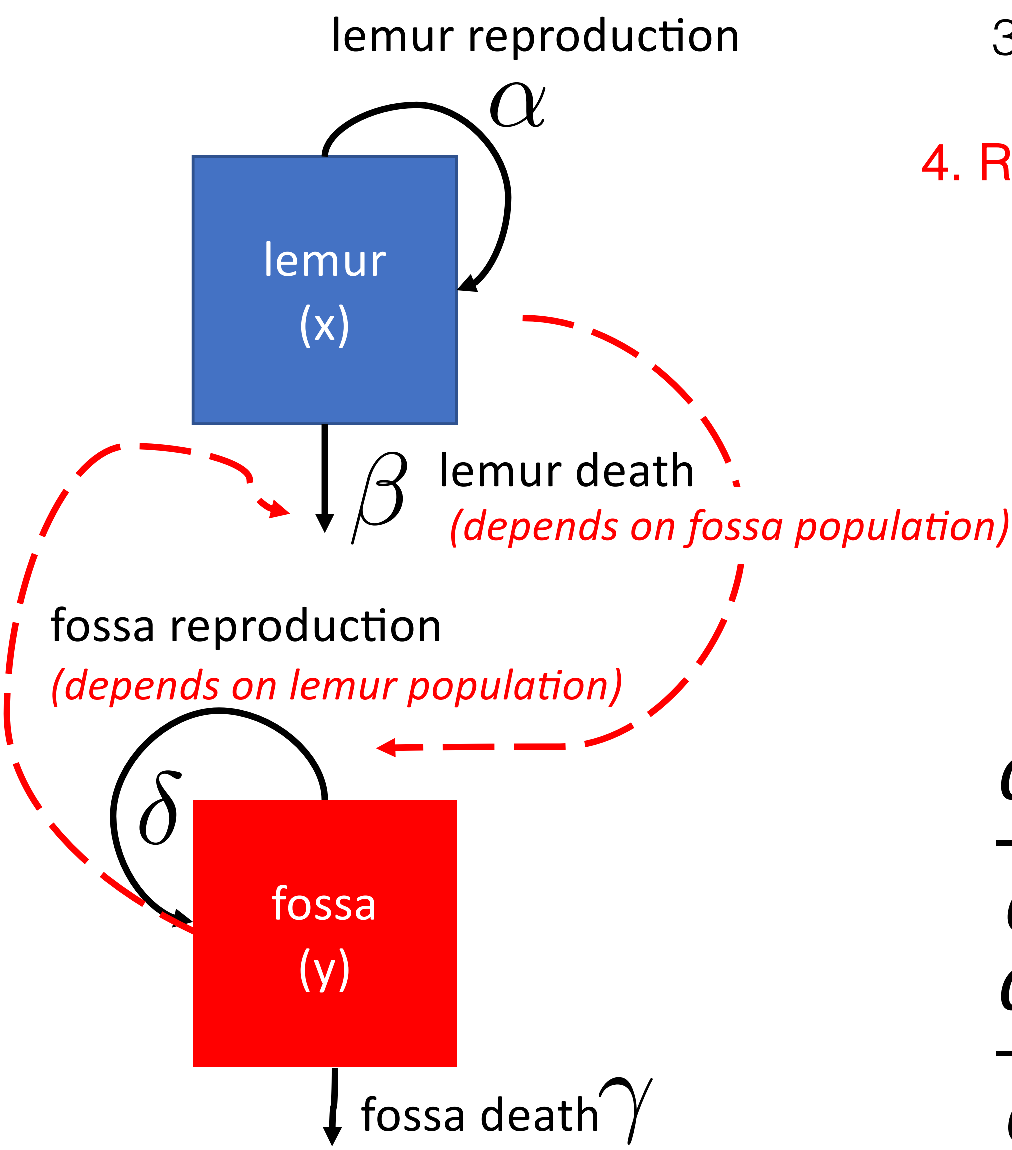

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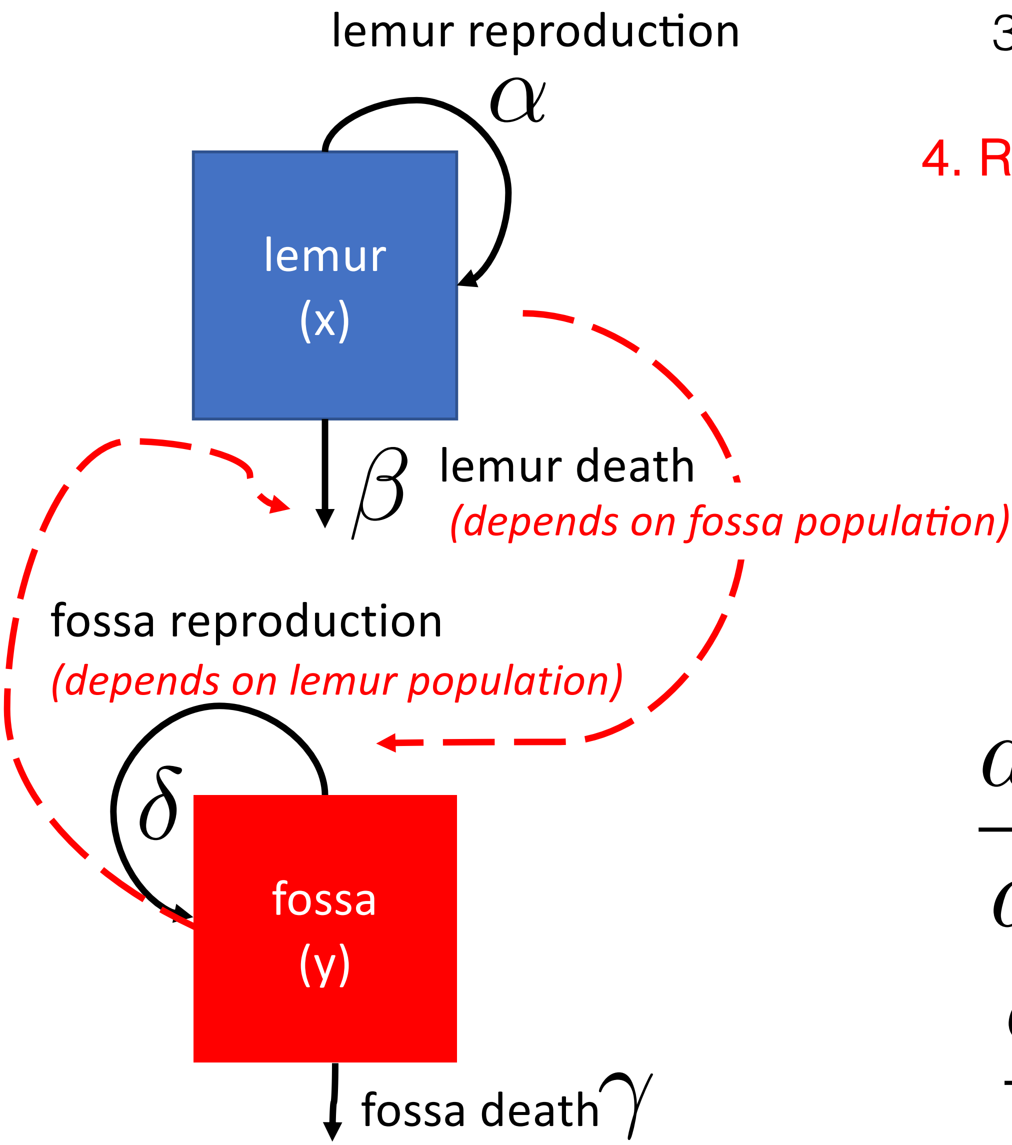

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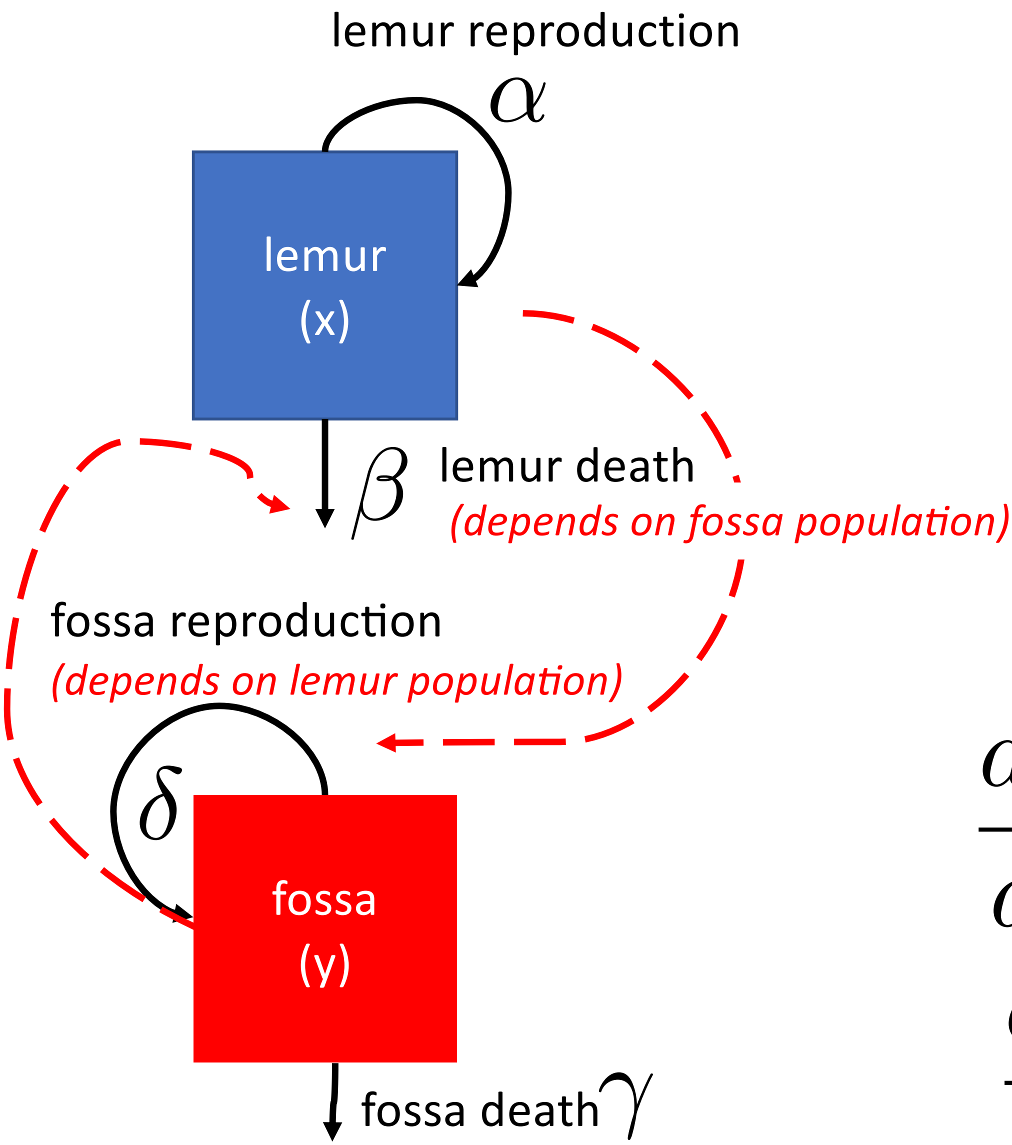
The predator-prey model

Practical tip

Set the size of the lemur, or fossa population to 0 to check that the model makes sense.



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Good! 🙂 ✅ ❤️ 👍

Bad! 👎 😮 ❌ 🤯

(or ask a question in the chat)

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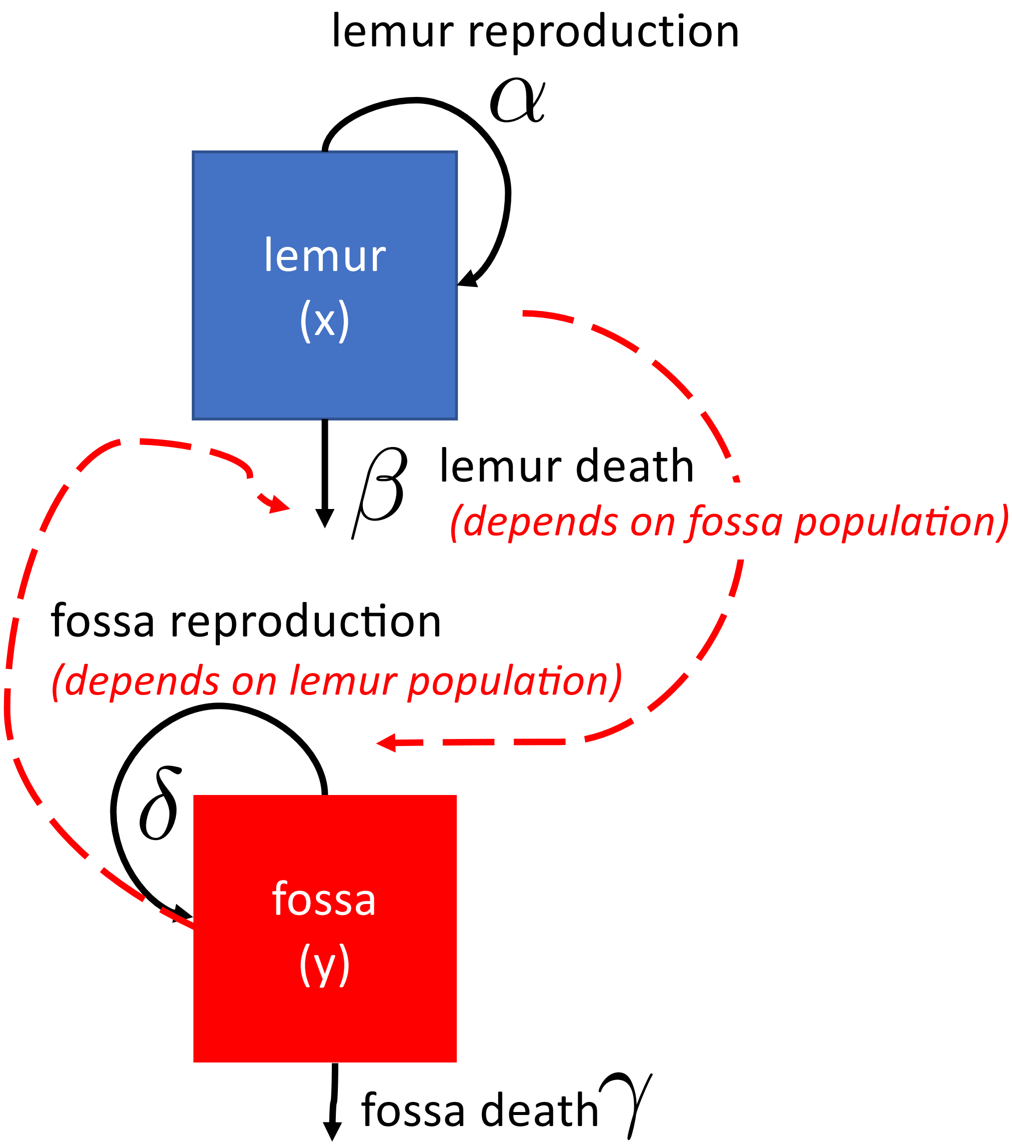

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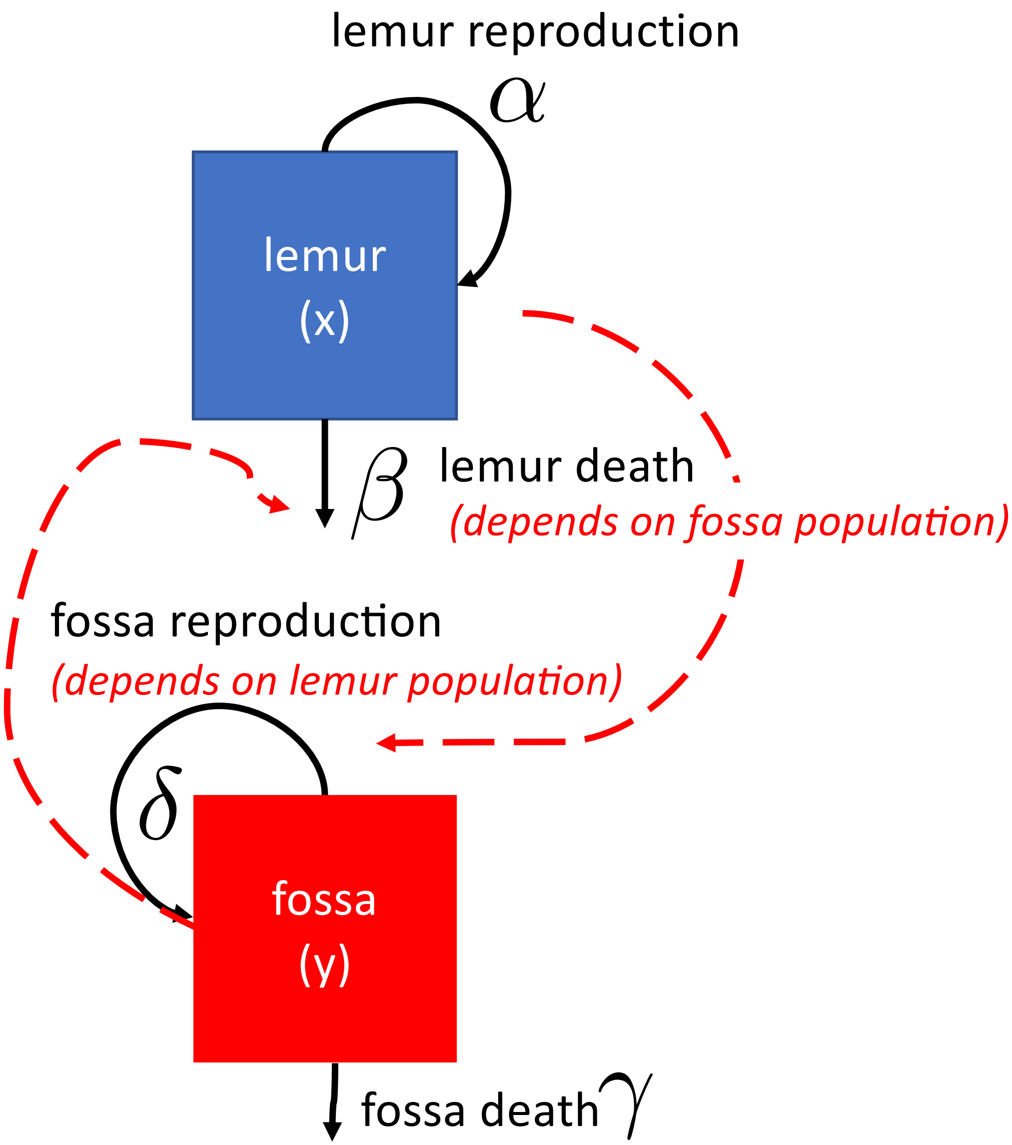

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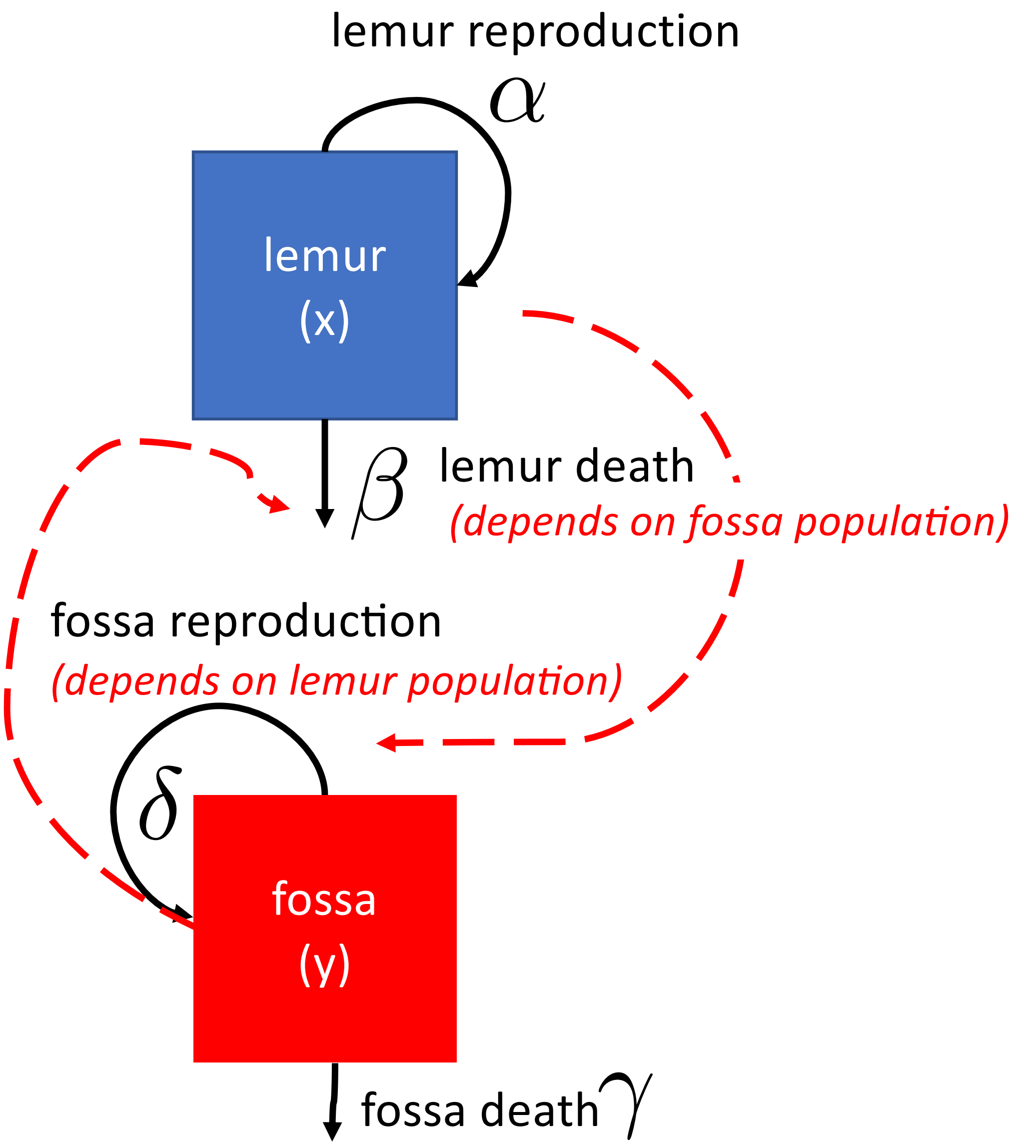

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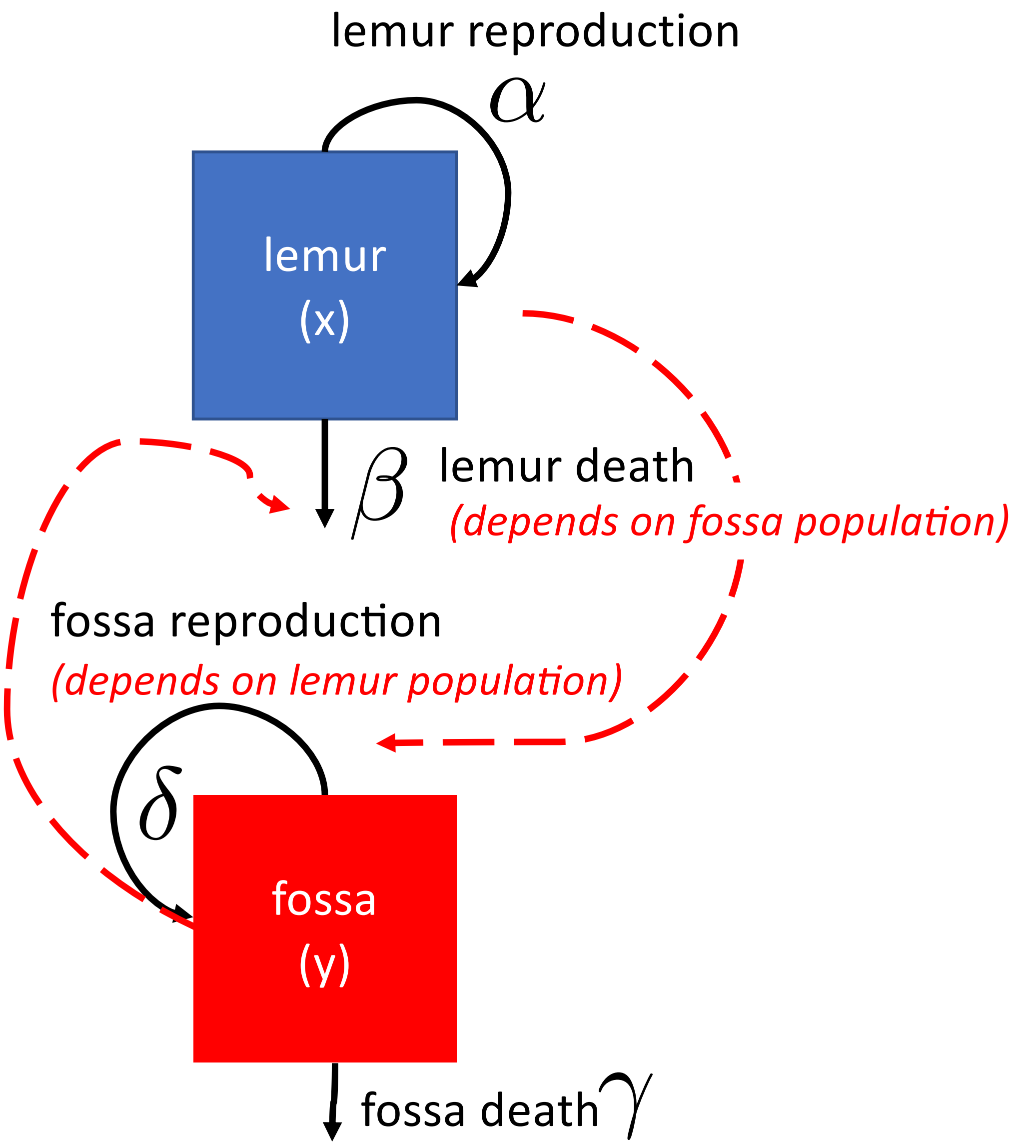

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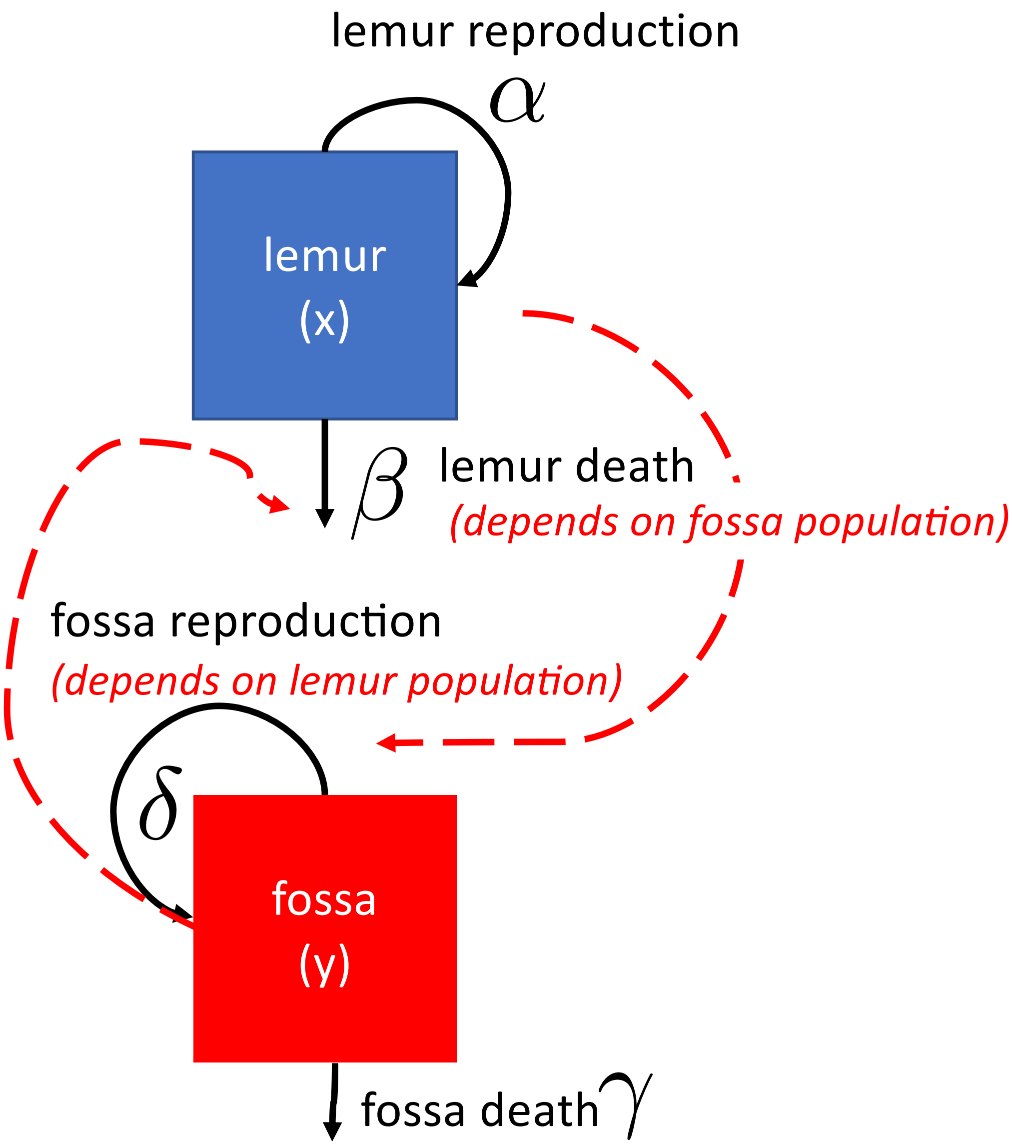

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$$\frac{dx}{dt} = x\alpha \rightarrow x(t) = x_0 e^{\alpha t}$$

$$\frac{dy}{dt} = 0$$

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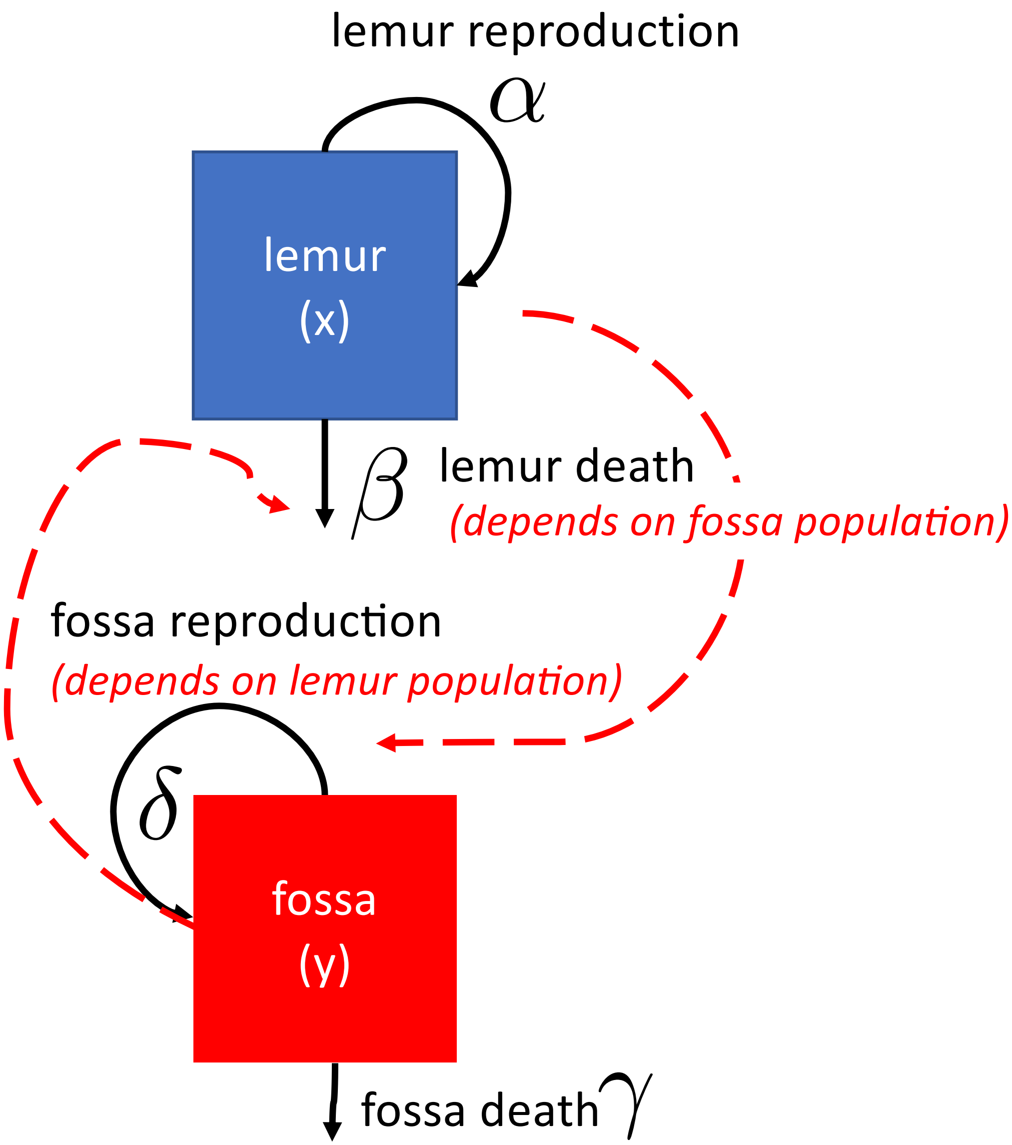

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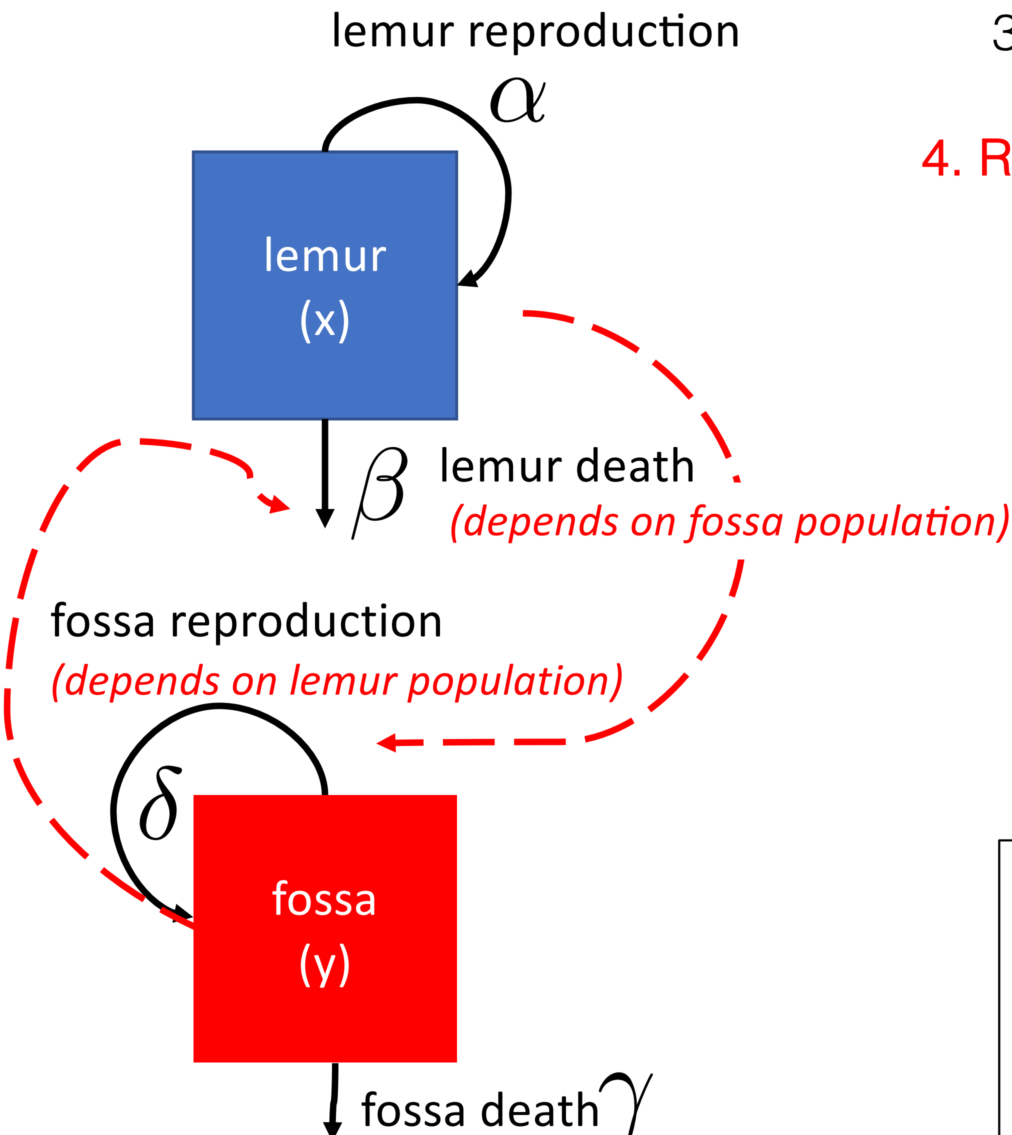
When there are no fossa:

- The lemur population grows exponentially
- The fossa population does not grow

The predator-prey model



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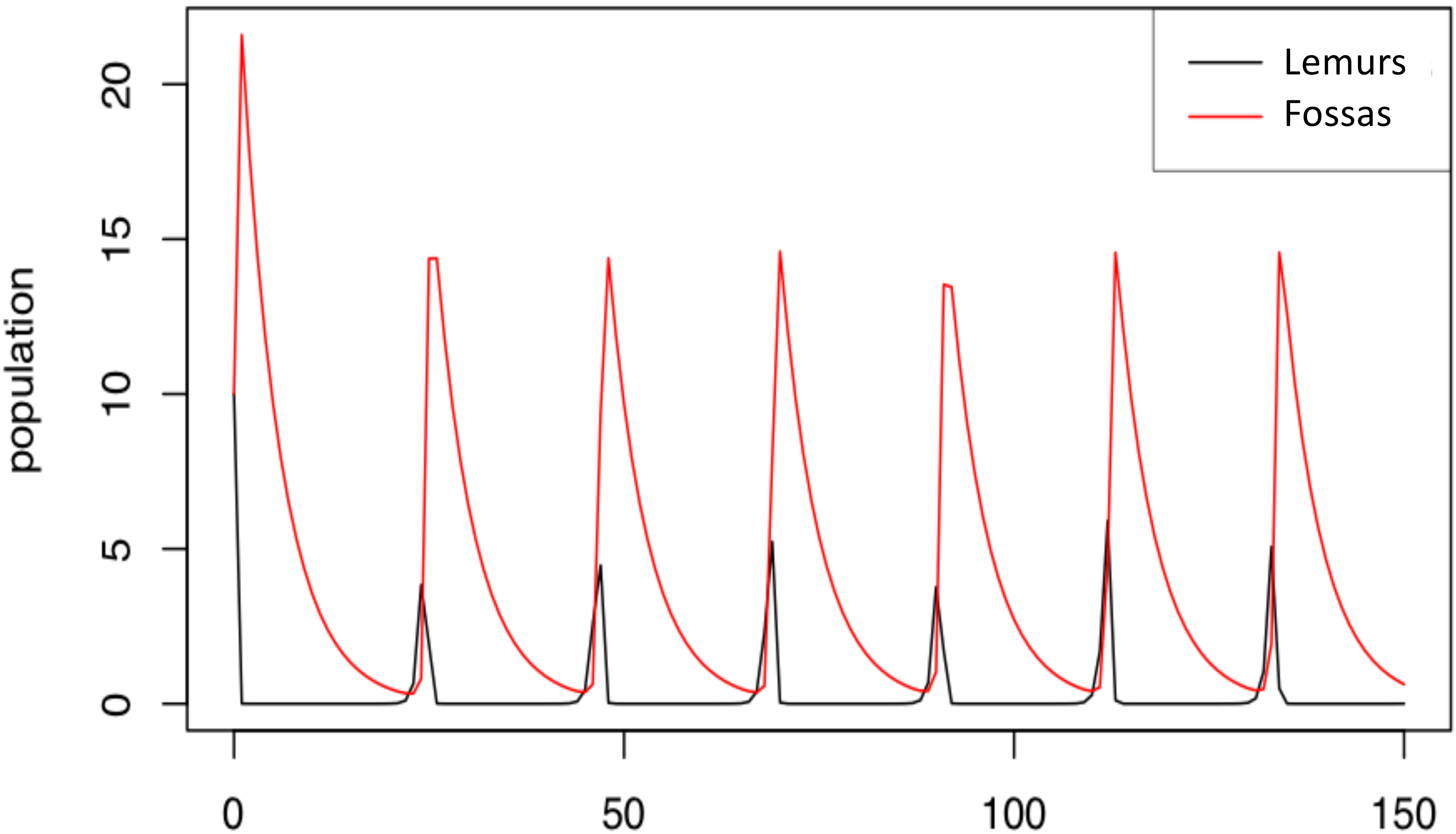
SOME ASSUMPTIONS

- the **lemur** has an unlimited food supply
- the **lemur** only dies from being eaten by fossa
- the **fossa** is totally dependent on a single prey species (the lemur) as its only food supply

The predator-prey model



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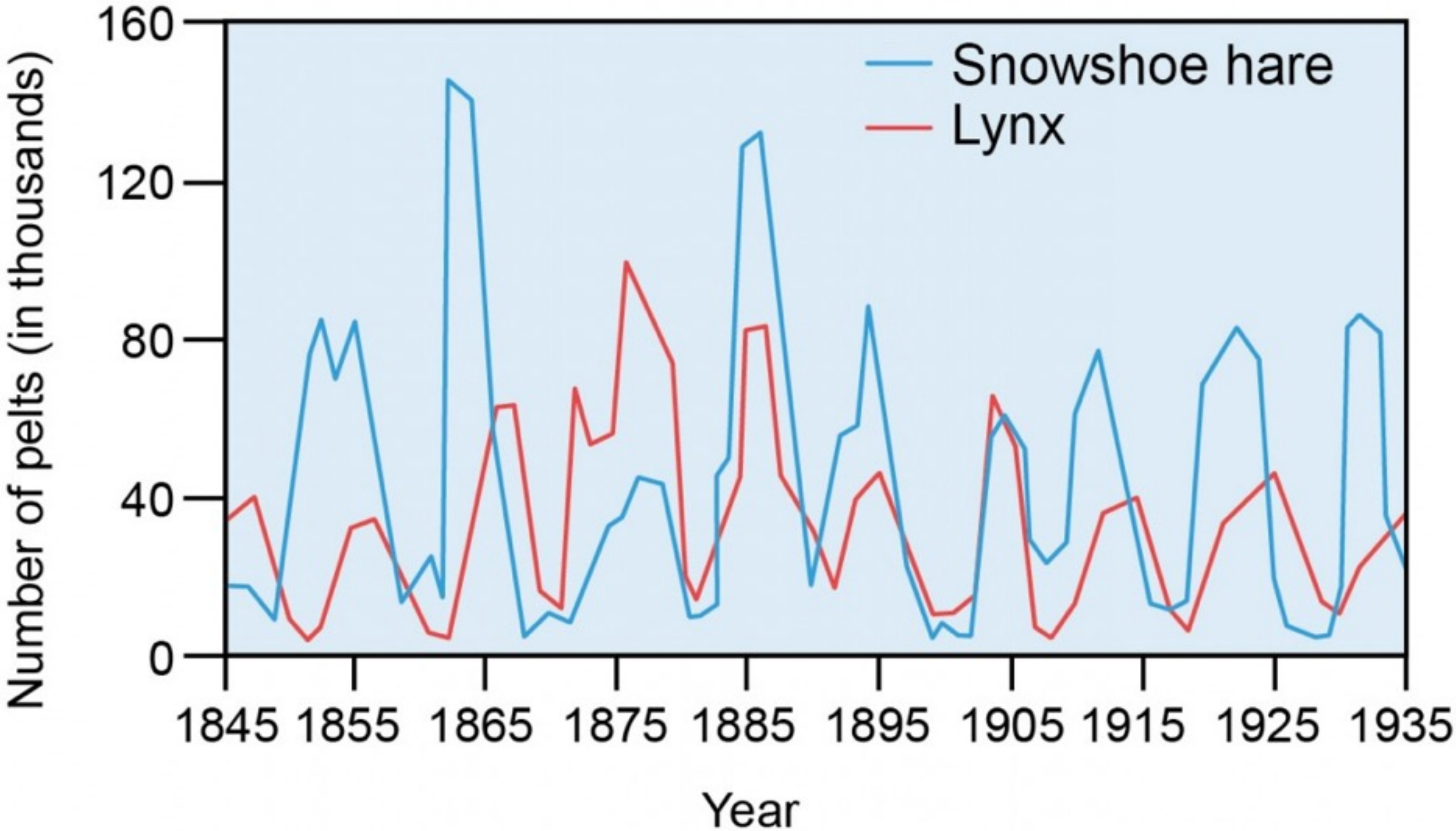

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Good! 🙂 ✅ ❤️ 👍

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(or ask a question in the chat)

4. SIR Models
4. Les modèles SIR

The SIR model

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How does measles transmit through Antananarivo?

Comment la rougéole se transmet-elle à Antananarivo?

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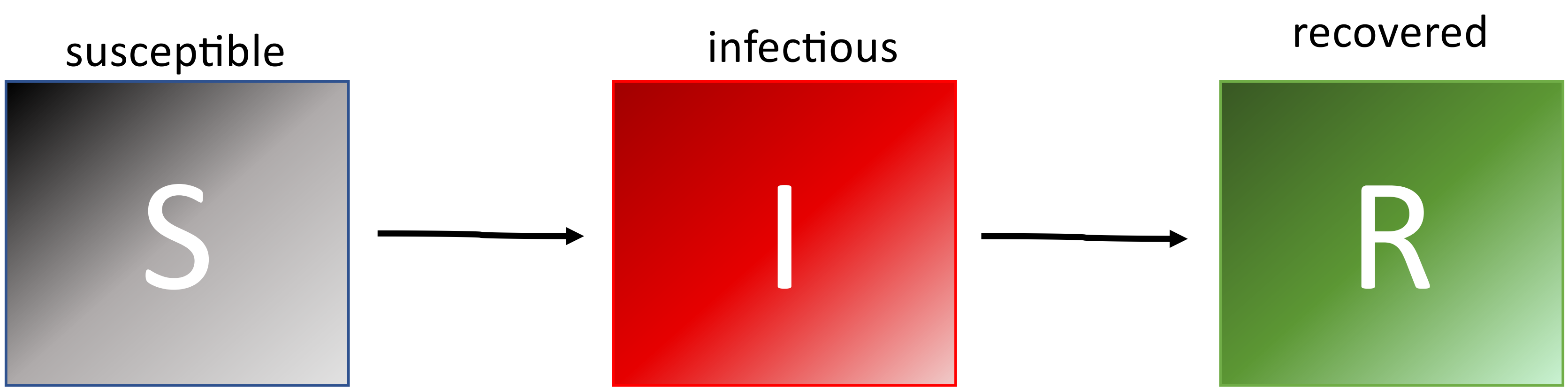

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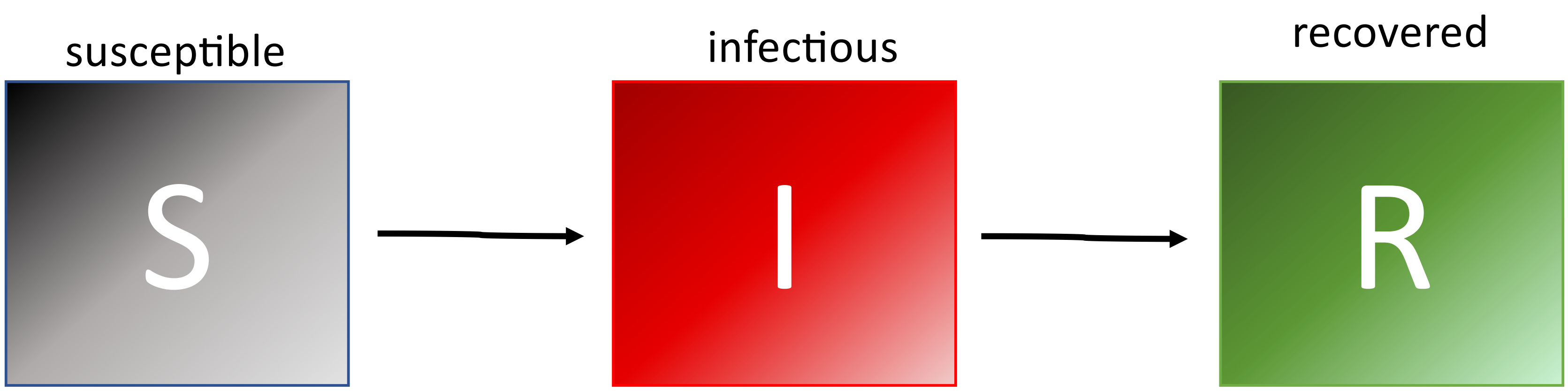

What are the big assumptions here?

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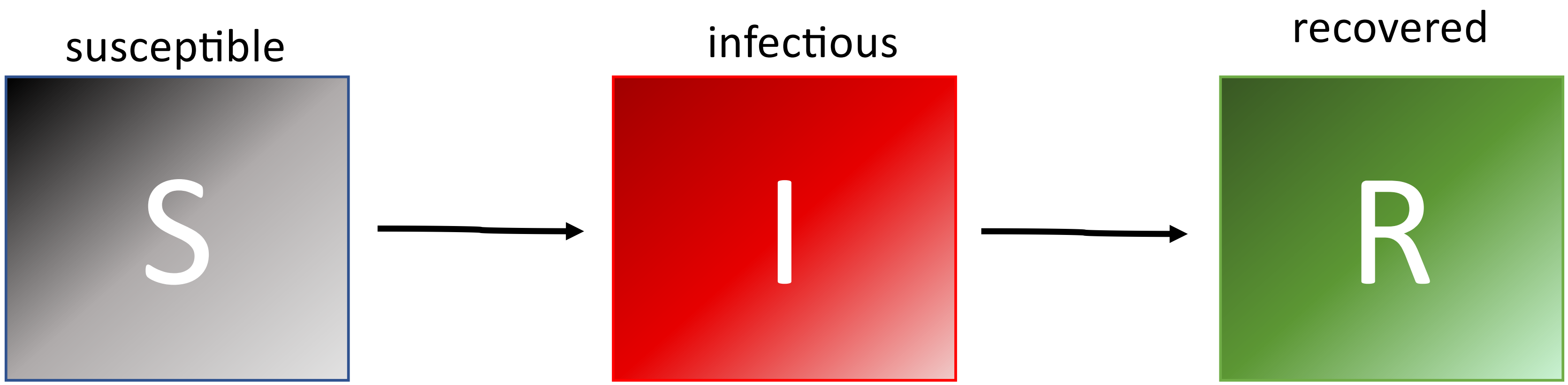
everyone is either:



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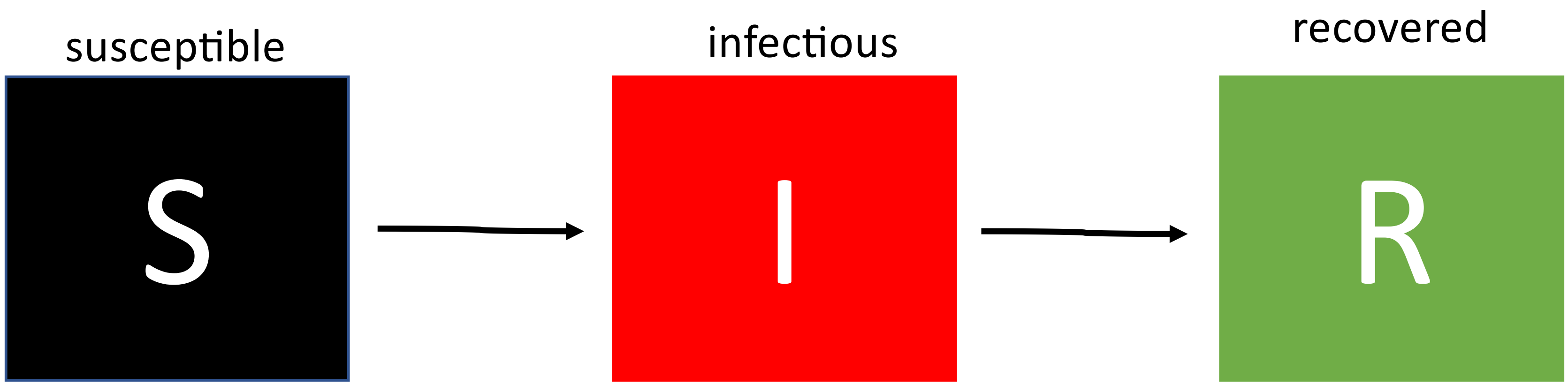

people mix uniformly (mass action)

les gens se mélangent uniformément

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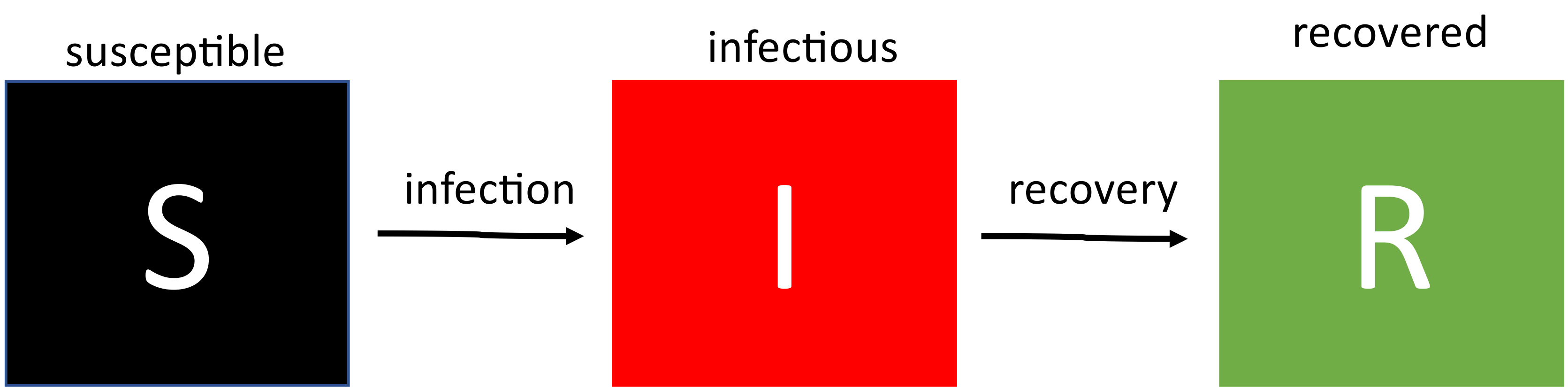

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(infectious when infected)
pas de période de latence

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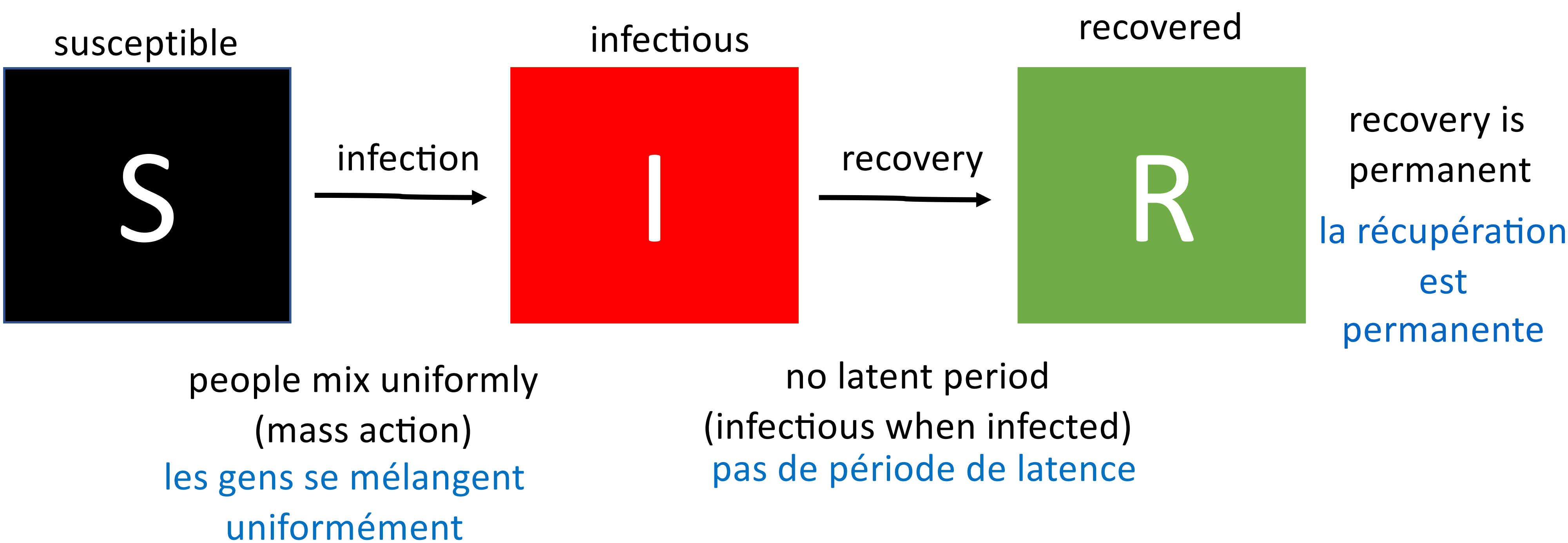

The SIR model

population size constant - no births, deaths, or migration

taille de population constante

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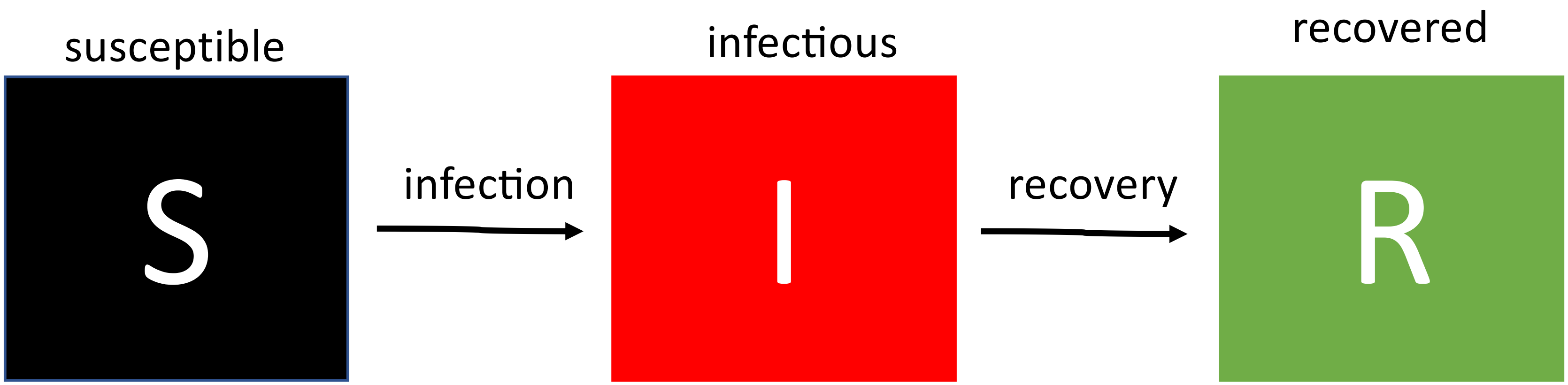

recovery is permanent

la récupération est permanente

people mix uniformly (mass action)

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The SIR model

Parameters

β: transmission rate

γ: rate of recovery

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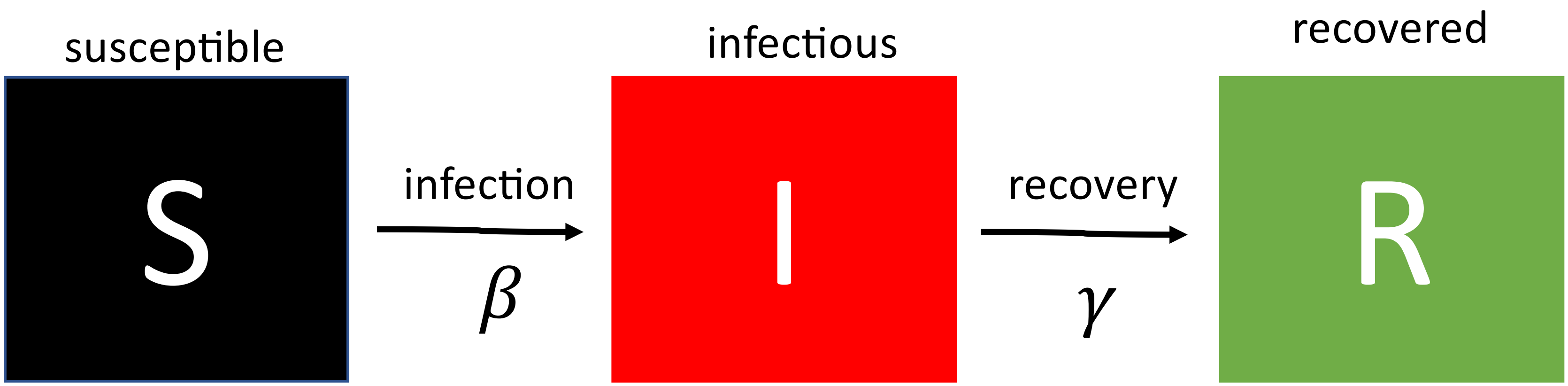

$$\frac{dS}{dt} = [\text{ rate of flow into S }] - [\text{ rate of flow out of S }]$$

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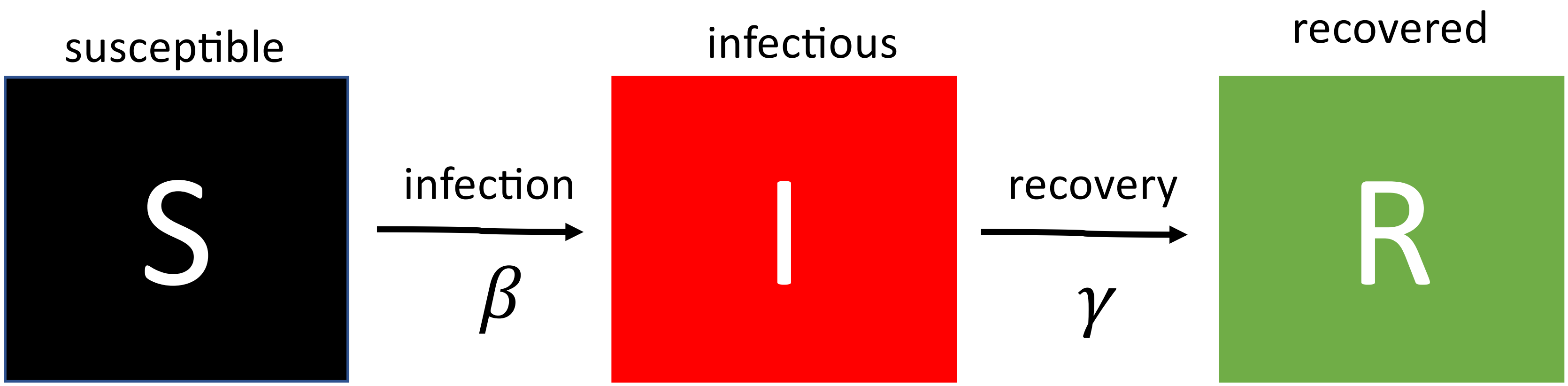

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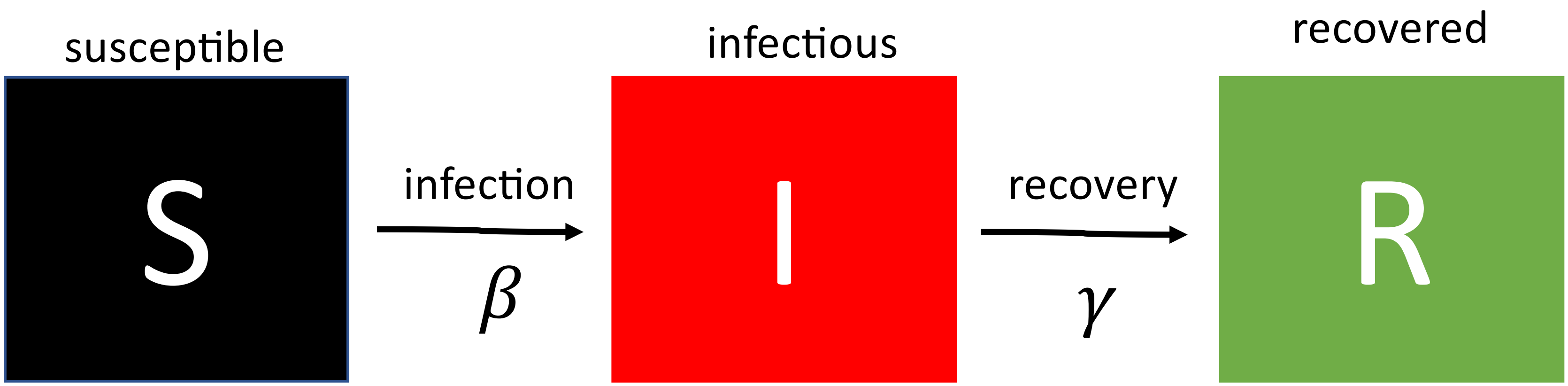

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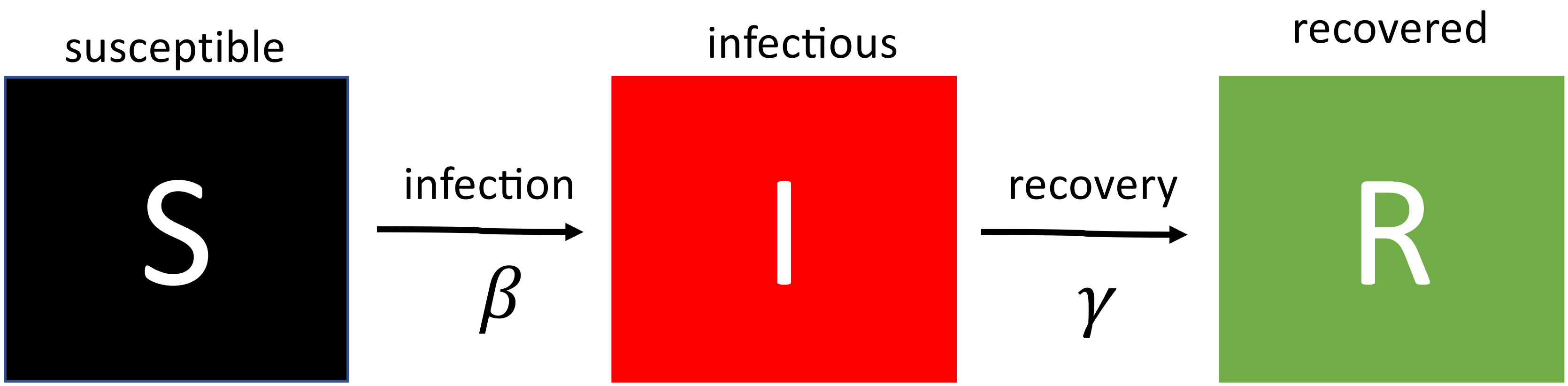

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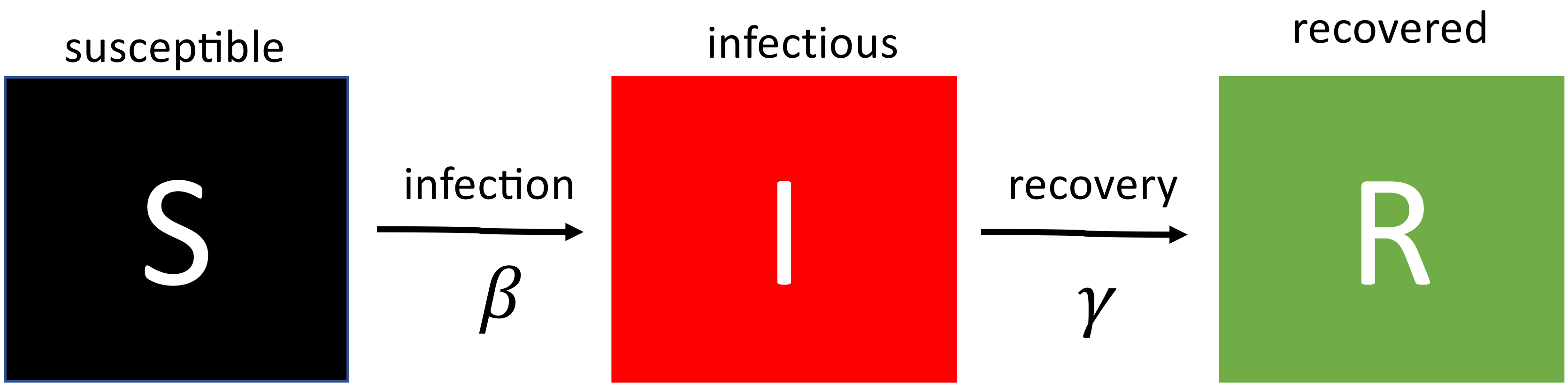

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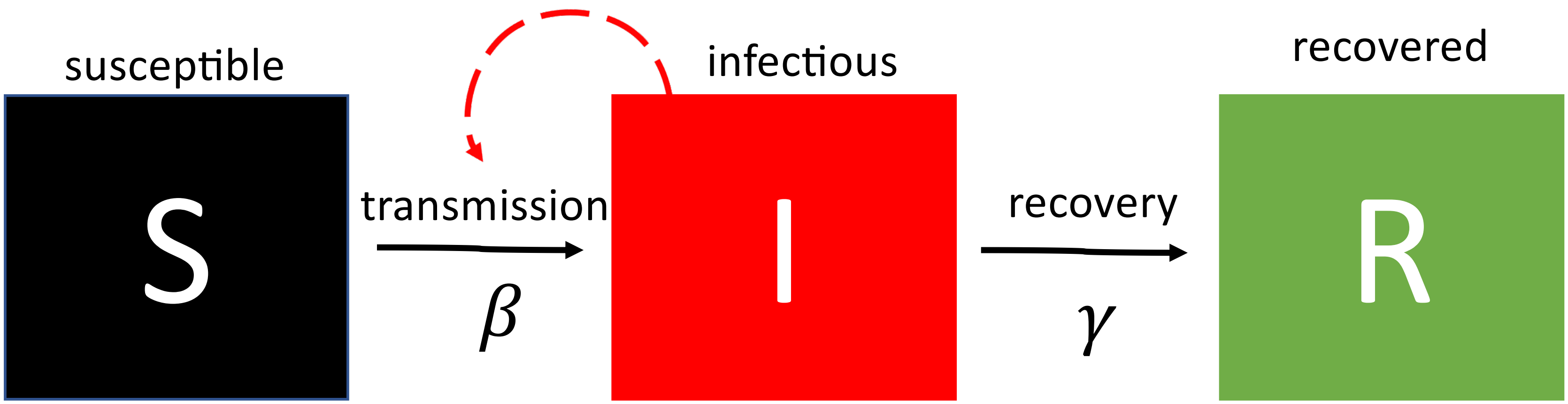
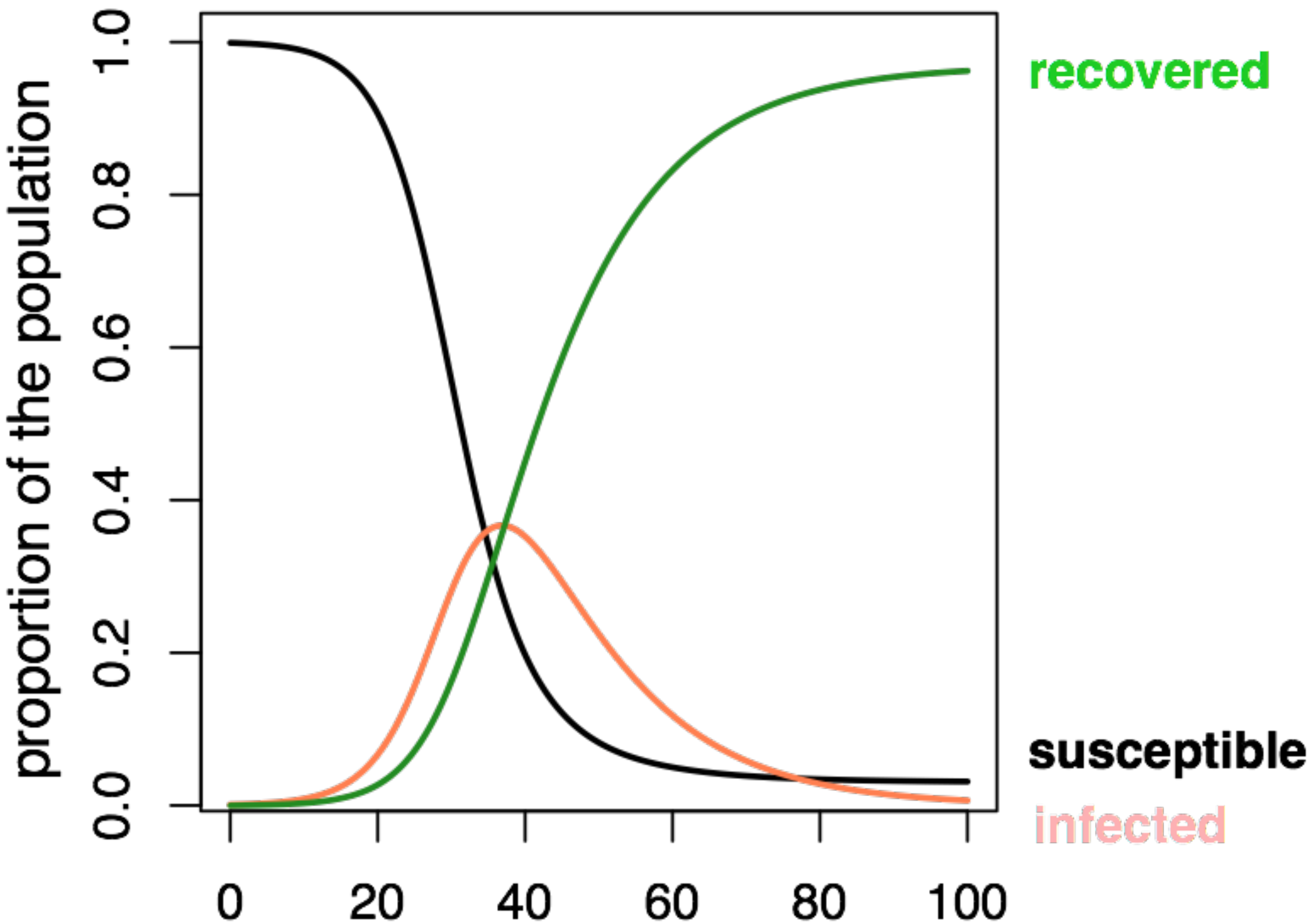
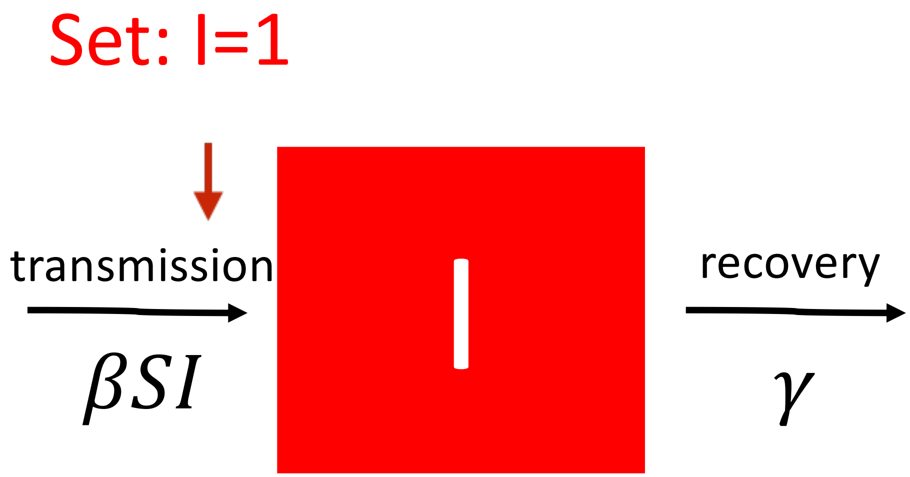


What will the dynamics look like?

The SIR model



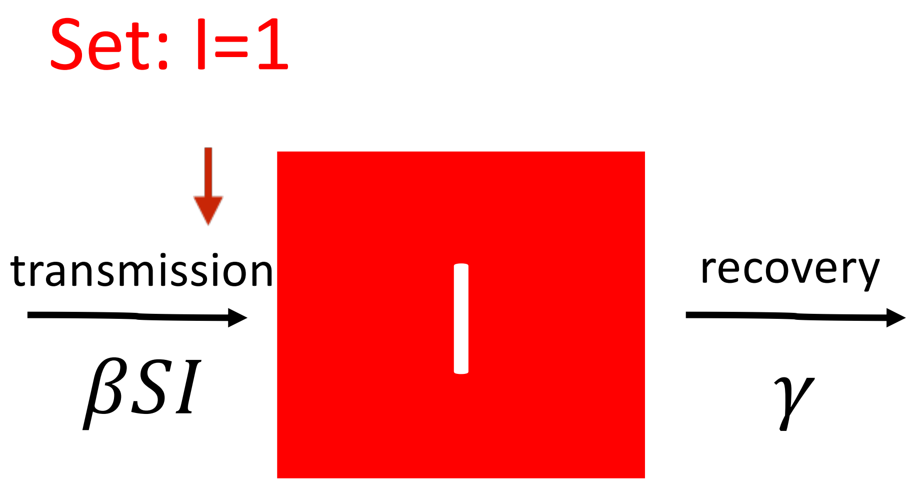
The SIR model

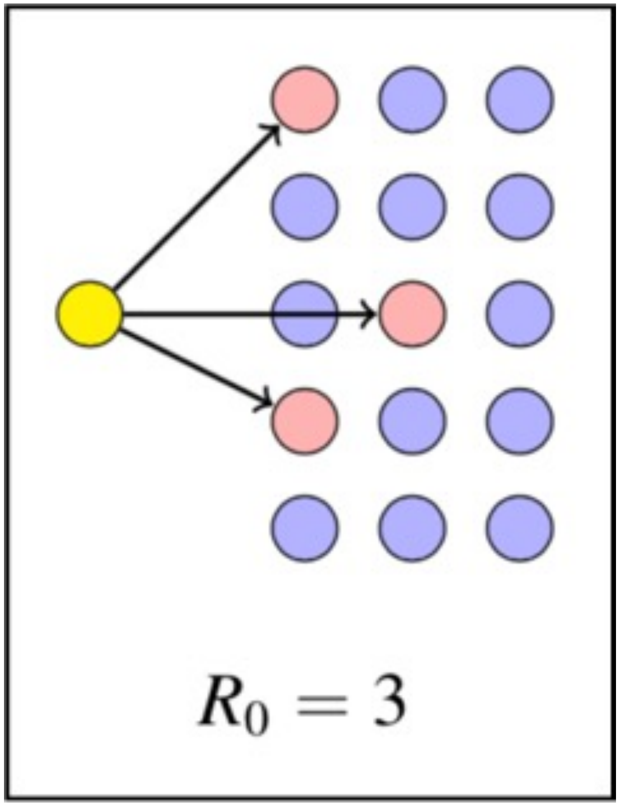


$$R_0 = \beta N / \gamma$$

The average number of persons infected by an infectious individual when everyone is susceptible (S=100%, or S=1, start of an epidemic)

The SIR model





$$R_0 = \beta N/\gamma$$

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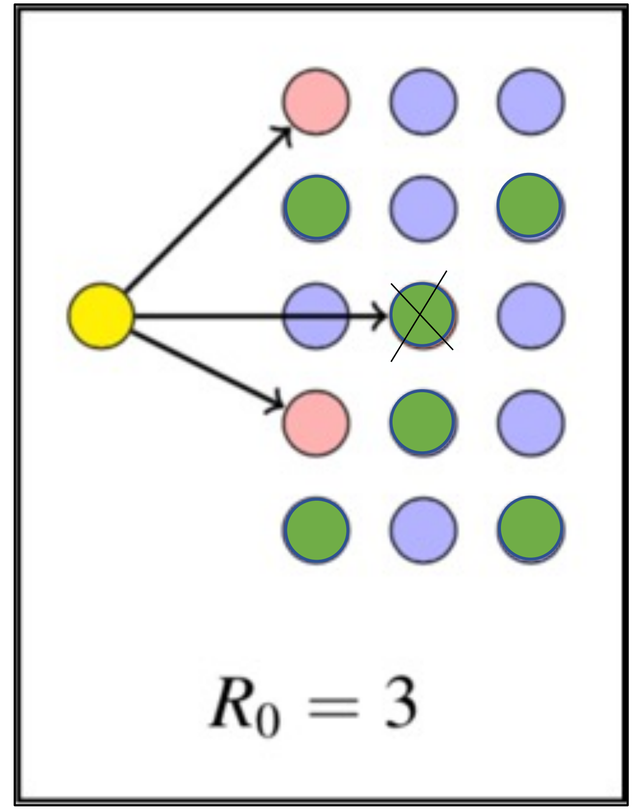


$$R_0 = \beta N / \gamma$$

The average number of persons infected by an infectious individual when everyone is susceptible (S=100%, or S=1, start of an epidemic)

$$R_E = R_0 S \quad \text{“R-effective"}$$

…as the epidemic progresses and S falls

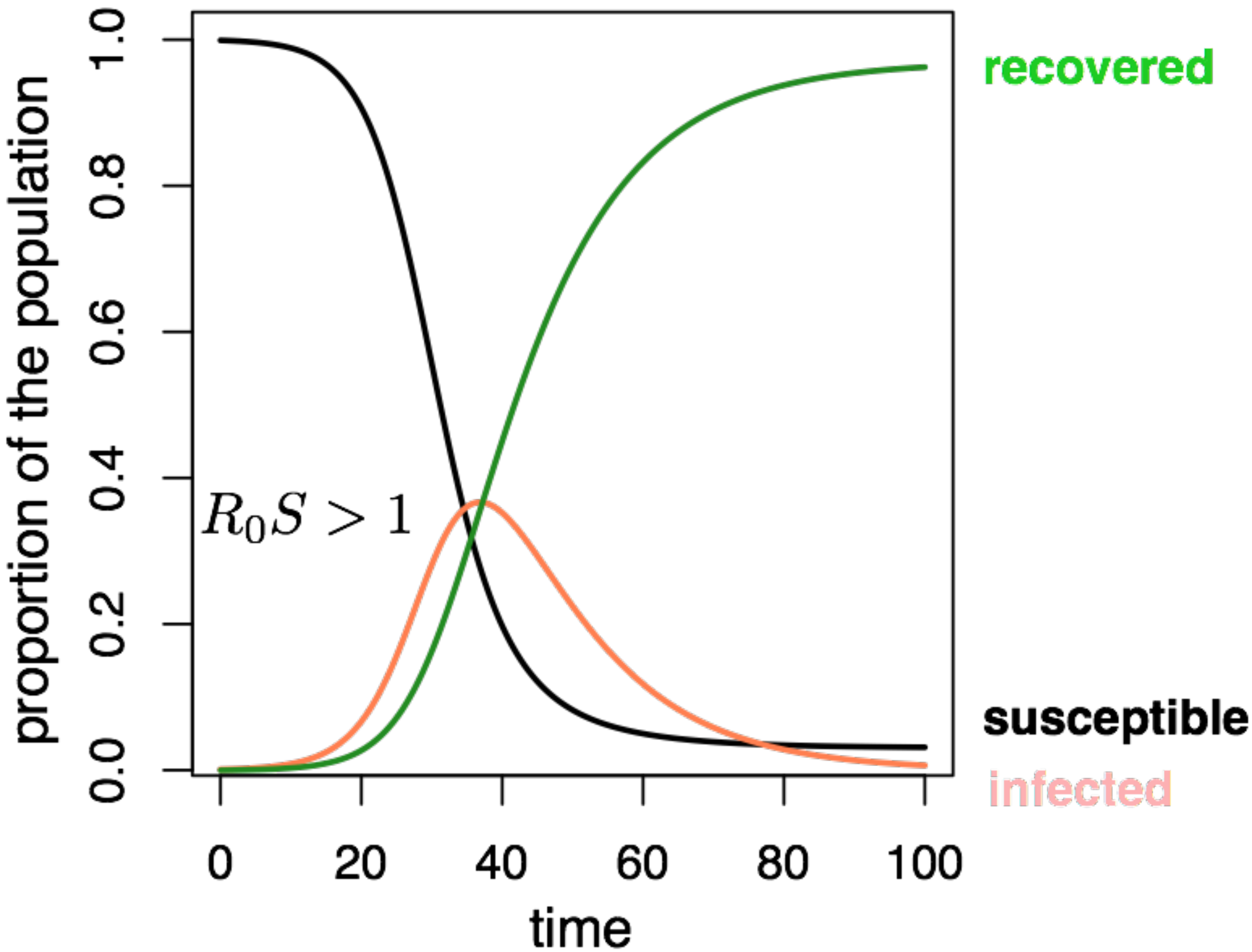
Question:

What is the minimum R0 value at which an epidemic can grow?

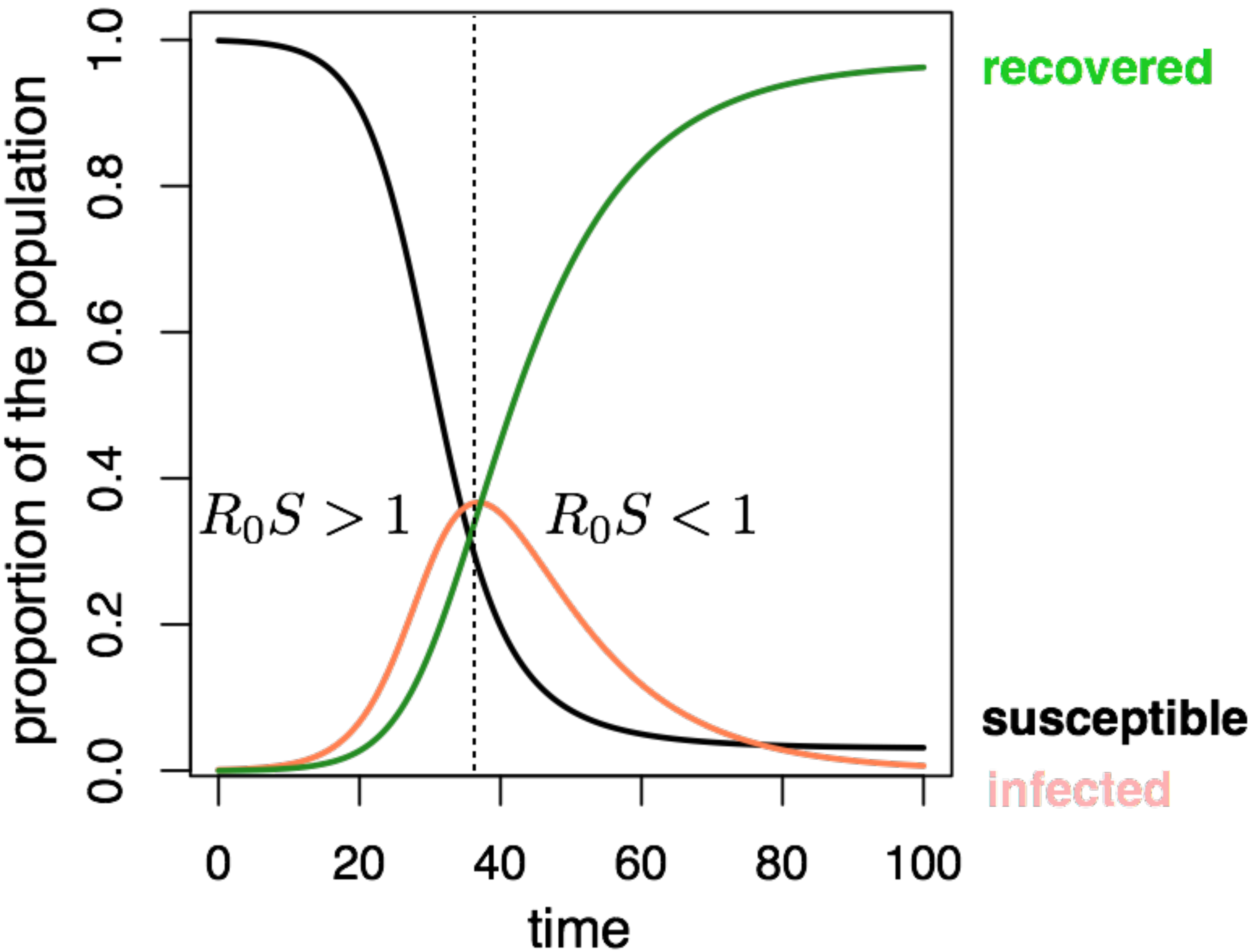
Quelle est la valeur R0 minimale à laquelle une épidémie peut se augmenter ?

(Type answers in the chat)

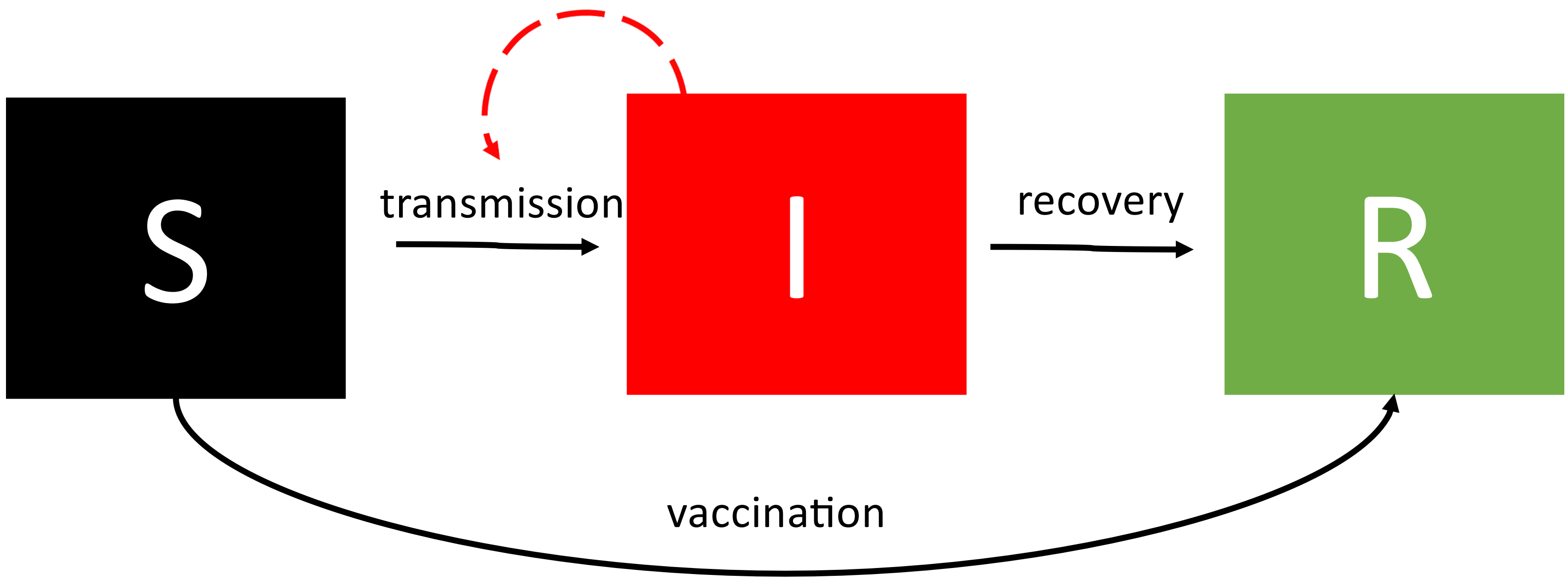
The SIR model



The SIR model



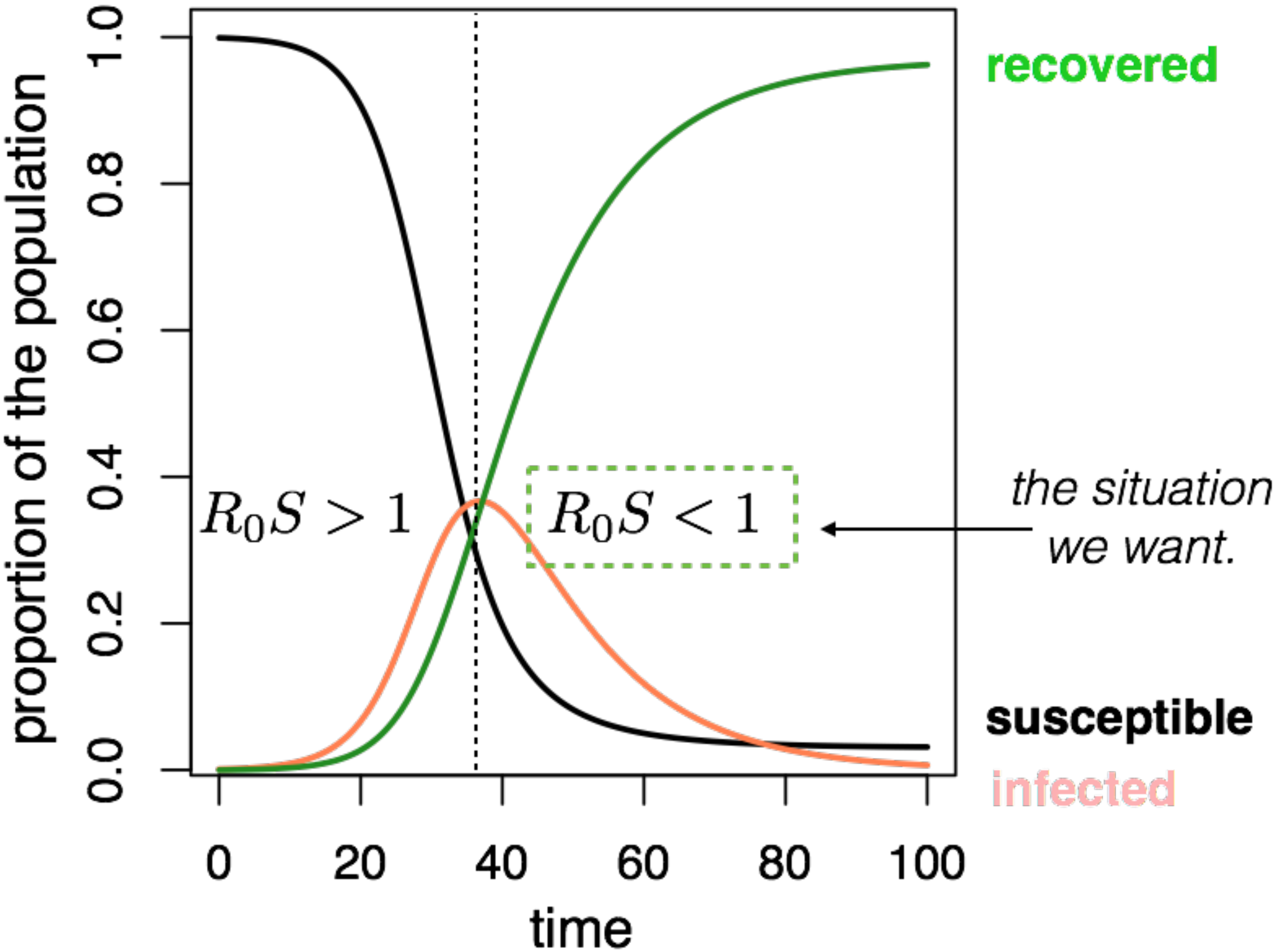
The SIR model : vaccination



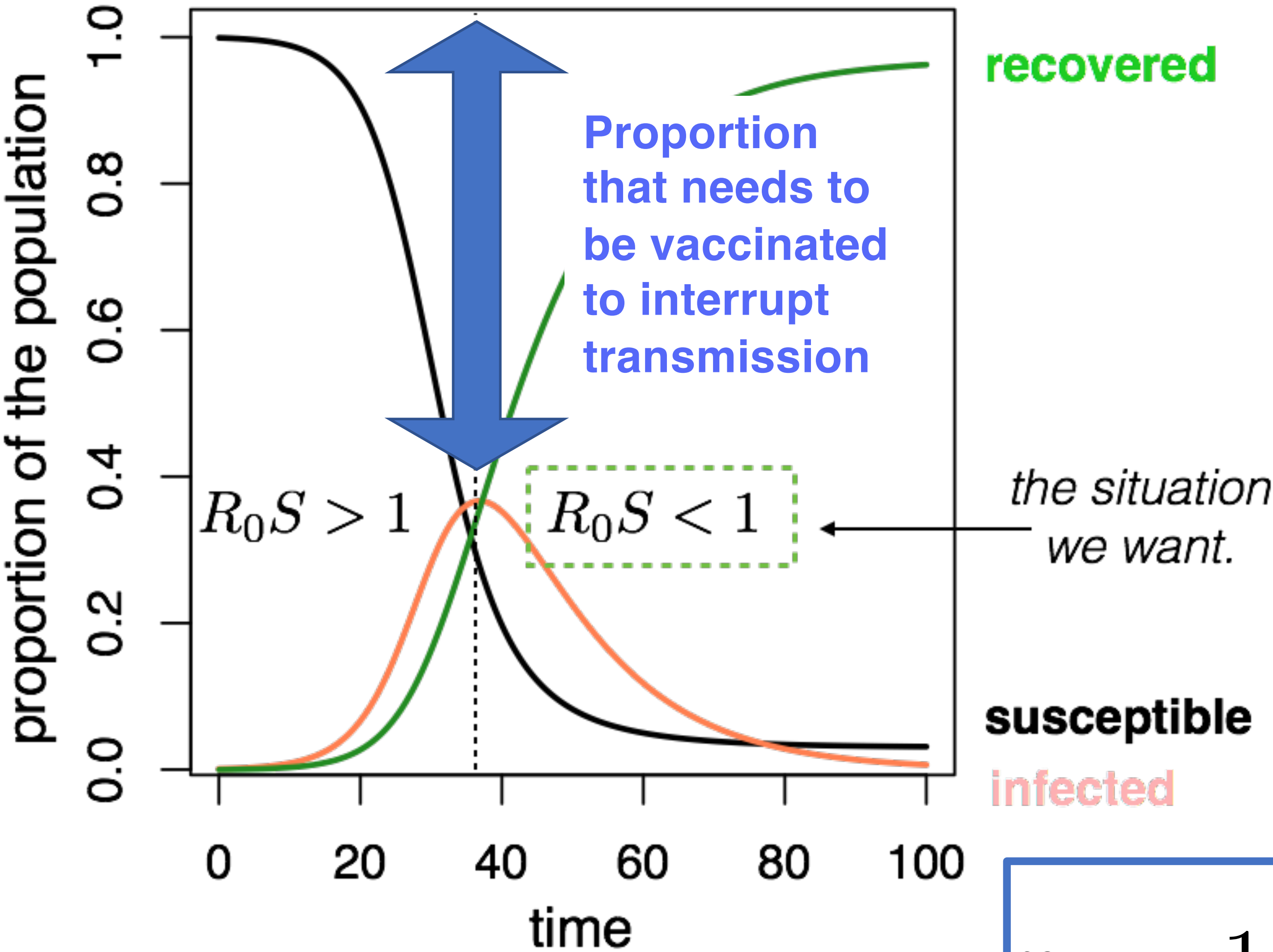
Vaccination moves people out of susceptibles into the immune (recovered) class.

La vaccination éloigne les personnes sensibles de la maladie dans la classe immunitaire (rétablie).

The SIR model : vaccination



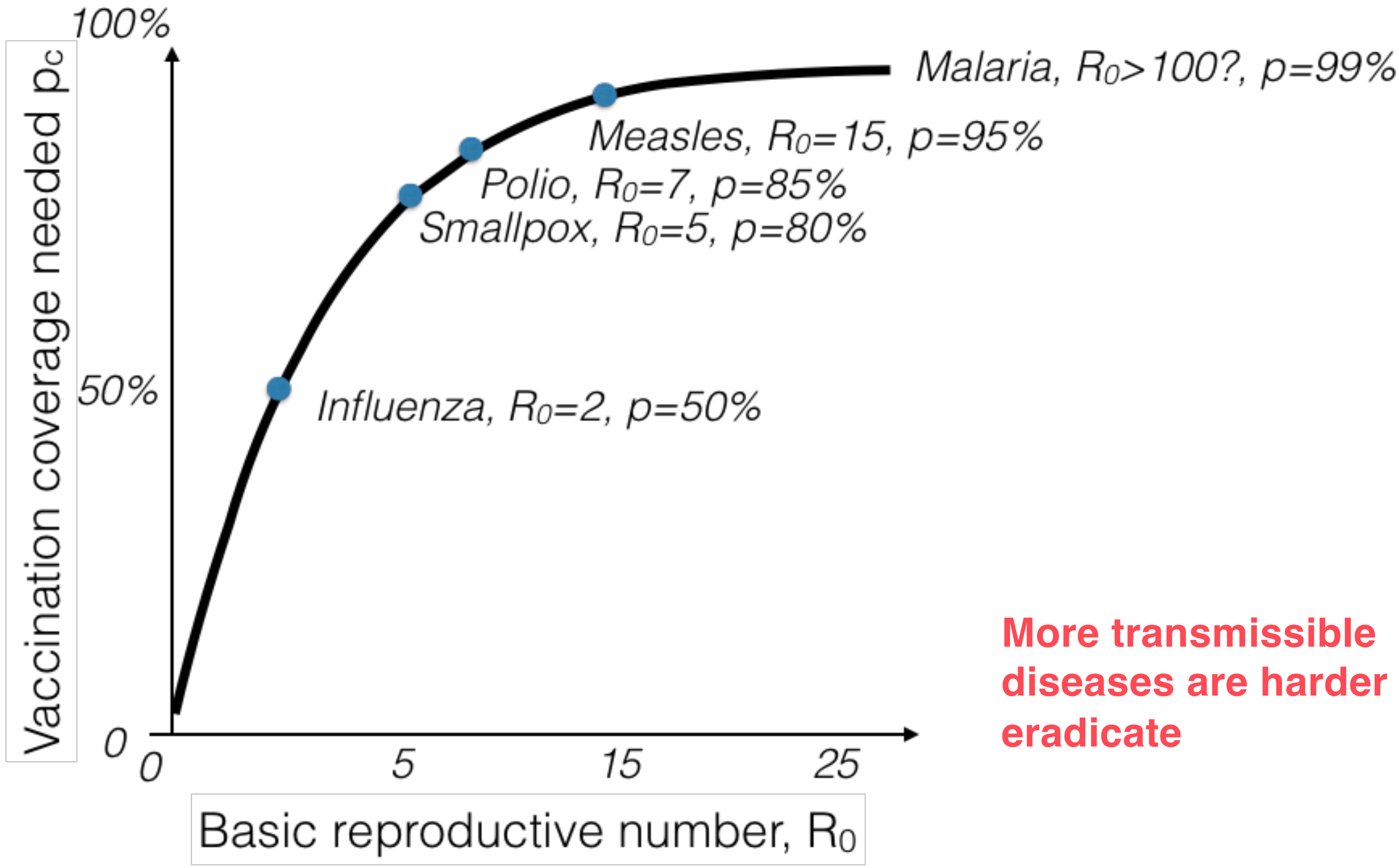
The SIR model : vaccination



$$p_c = 1 - \frac{1}{R_0}$$

The SIR model : eradication

$$p_c = 1 - \frac{1}{R_0}$$



More transmissible diseases are harder to eradicate

How are we doing?

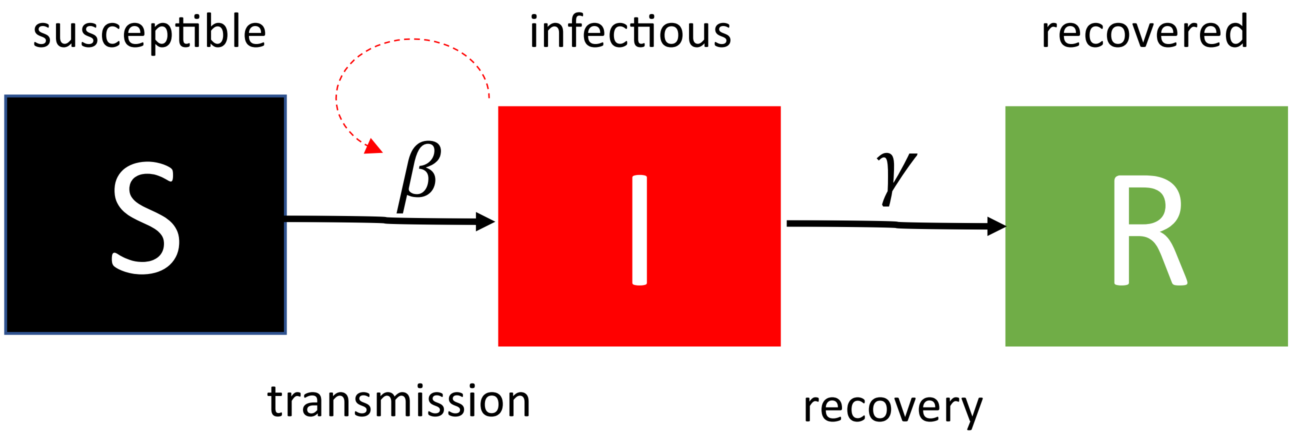
Good! 🙂 ✅ ❤️ 👍

Bad! 👎 😮 ❌ 🤯

(or ask a question in the chat)

5. Modifying SIR models

Modifying the SIR model → Include a latent period SEIR

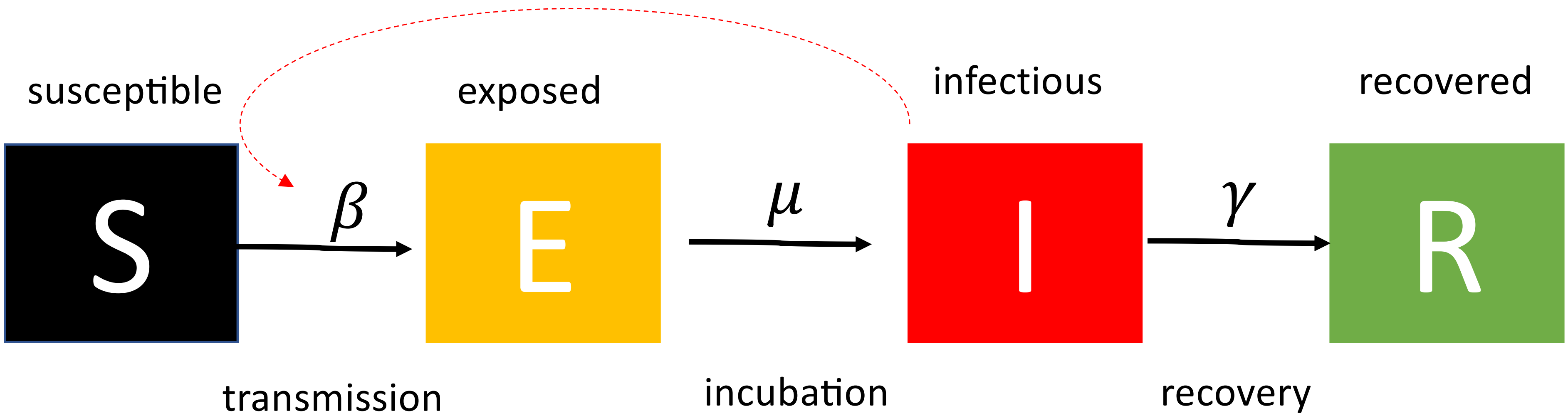


$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = \beta SI - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

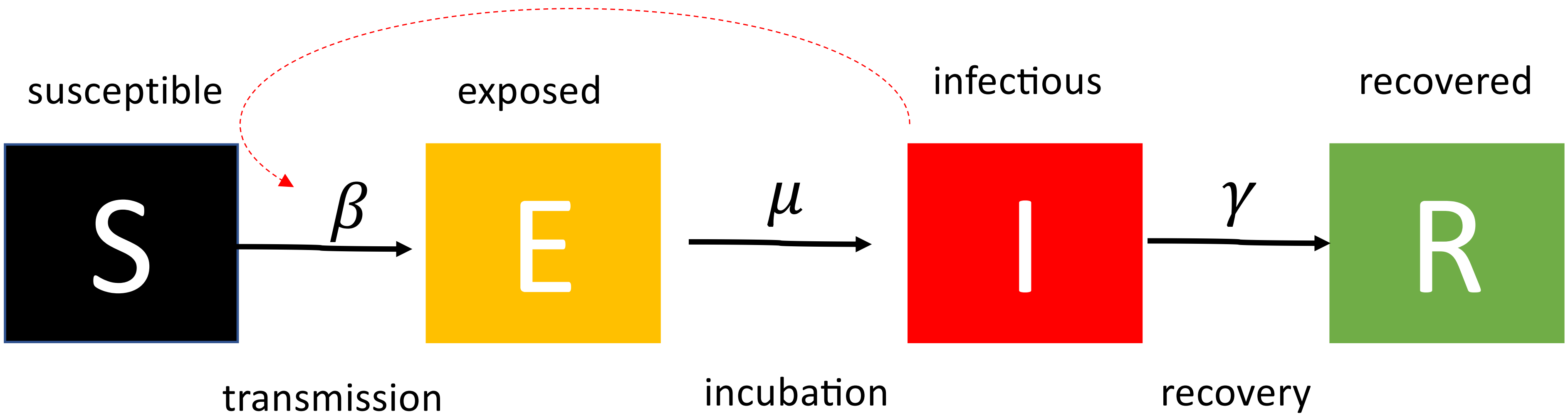
Modifying the SIR model → Include a latent period SEIR



$$\frac{dS}{dt} = -\beta SI$$
$$\frac{dI}{dt} = \beta SI - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

← How do we modify the equations?

Modifying the SIR model → Include a latent period SEIR



$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dE}{dt} = \text{[rate of flow in] – [rate of flow out]}$$

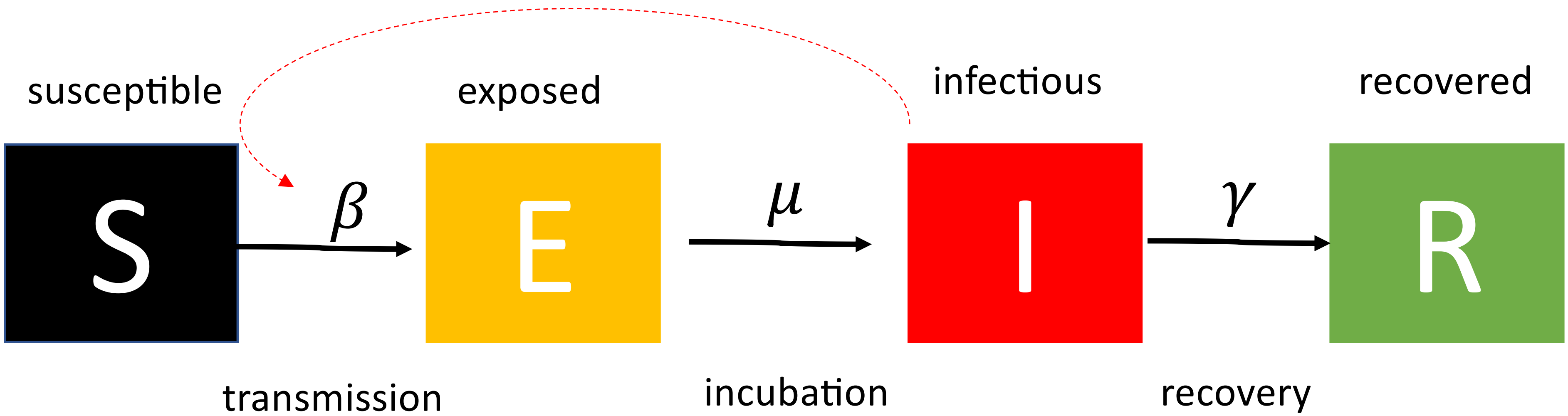
$$\frac{dI}{dt} = \beta SI - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

← How do we modify the equations?

1. Add a dE/dt equation

Modifying the SIR model → Include a latent period SEIR



$$\frac{dS}{dt} = -\beta S I$$

$$\frac{dE}{dt} = \beta S I - \mu E$$

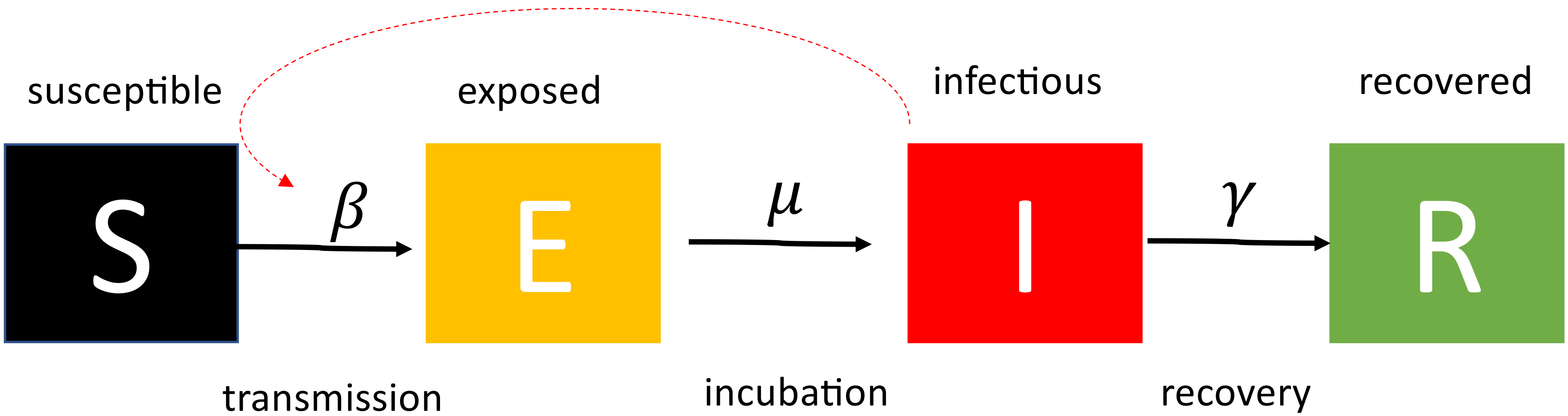
$$\frac{dI}{dt} = \cancel{\beta S I} - \gamma I \quad \text{[rate of flow in] – [rate of flow out]}$$

$$\frac{dR}{dt} = \gamma I$$

← How do we modify the equations?

1. Add a dE/dt equation
2. Check and update dI/dt equation.

Modifying the SIR model → Include a latent period SEIR

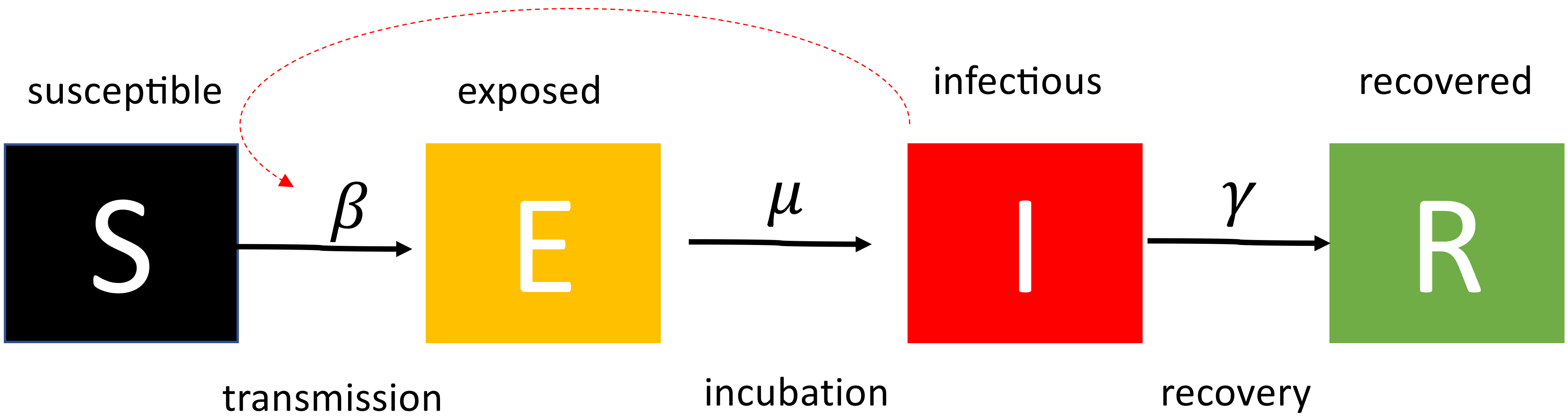


$$\frac{dS}{dt} = -\beta SI$$
$$\frac{dE}{dt} = \beta SI - \mu E$$
$$\frac{dI}{dt} = \mu E - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

← How do we modify the equations?

1. Add a dE/dt equation
2. Check and update the dI/dt equations.

Modifying the SIR model → Include a latent period SEIR



$$\frac{dS}{dt} = \boxed{-\beta S I}$$

$$\frac{dE}{dt} = \boxed{\beta S I} - \mu E$$

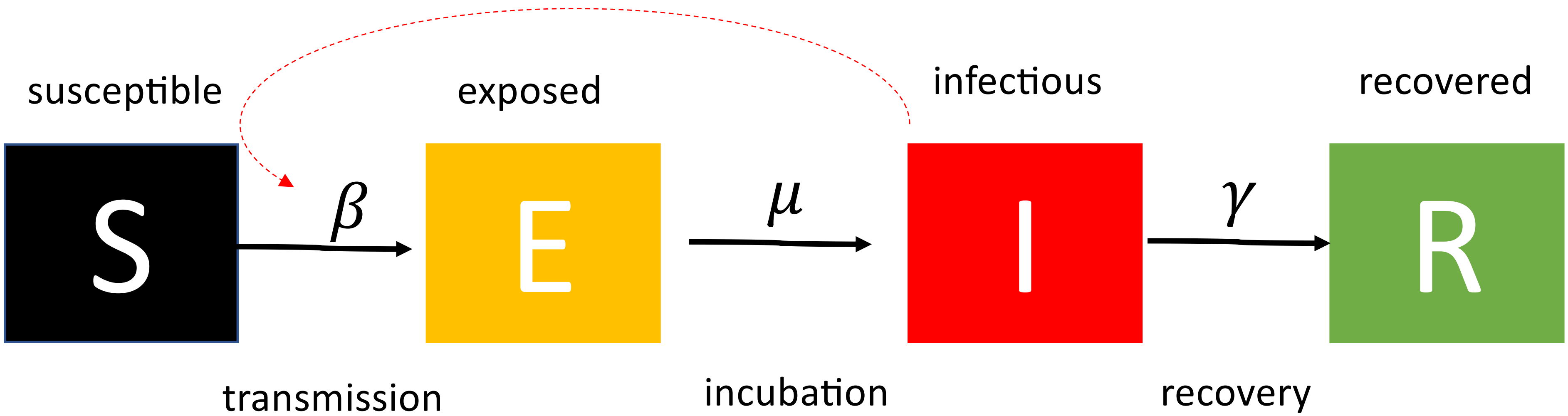
$$\frac{dI}{dt} = \mu E \; - \gamma I$$

$$\frac{dR}{dt} = \; \gamma I$$

← How do we modify the equations?

1. Add a dE/dt equation
2. Check and update the dS/dt and dI/dt equations.
3. Check that flows in are always equal to flows out

Modifying the SIR model → Include a latent period SEIR



$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dE}{dt} = \beta SI \boxed{- \mu E}$$

$$\frac{dI}{dt} = \boxed{\mu E} - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

← How do we modify the equations?

1. Add a dE/dt equation
2. Check and update the dS/dt and dI/dt equations.
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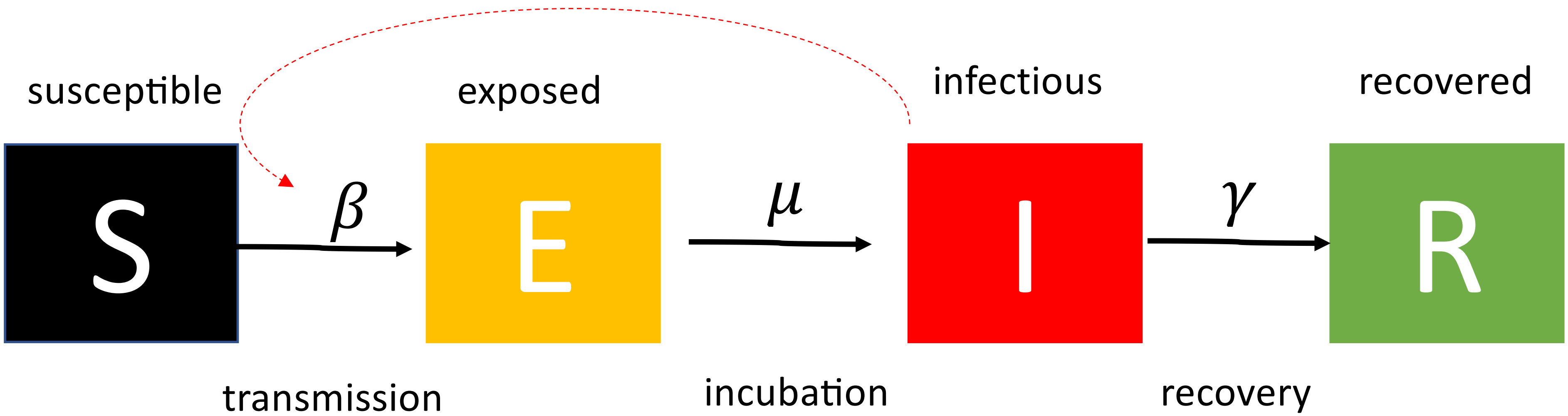
Modifying the SIR model → Include a latent period SEIR



$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dE}{dt} = \beta SI - \mu E$$

$$\frac{dI}{dt} = \mu E \boxed{- \gamma I}$$

$$\frac{dR}{dt} = \boxed{\gamma I}$$

← How do we modify the equations?

1. Add a dE/dt equation
2. Check and update the dS/dt and dI/dt equations.
3. Check that flows in are always equal to flows out

Key concepts

To modify a compartmental model:

1. Draw a box diagram.

2. Write down equations from the box diagram, carefully keeping track of flows into and out of each compartment.

3. Check to make sure the flows in are equal to the flows out.

How are we doing?

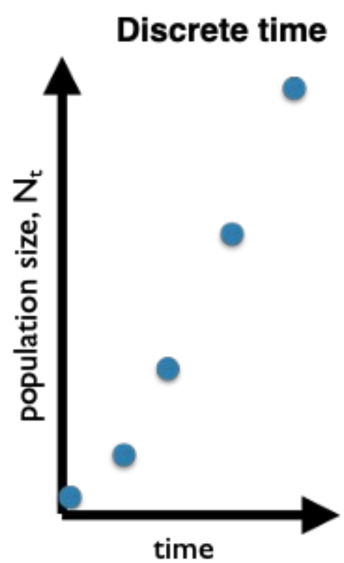
Good! 🙂 ✅ ❤️ 👍

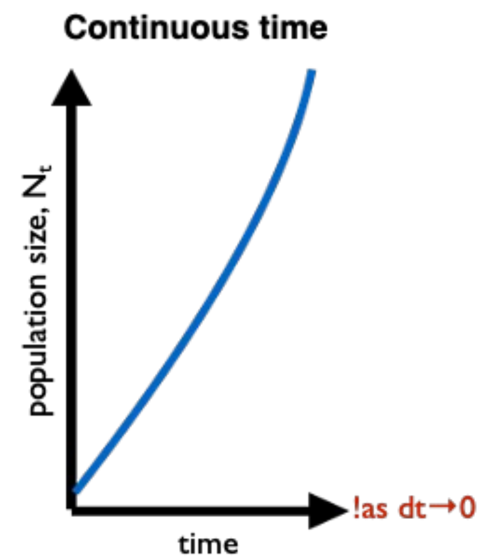
Bad! 👎 😮 ❌ 🤯

(or ask a question in the chat)

6. How do we calculate the state (dynamics) of the system from differential equations?

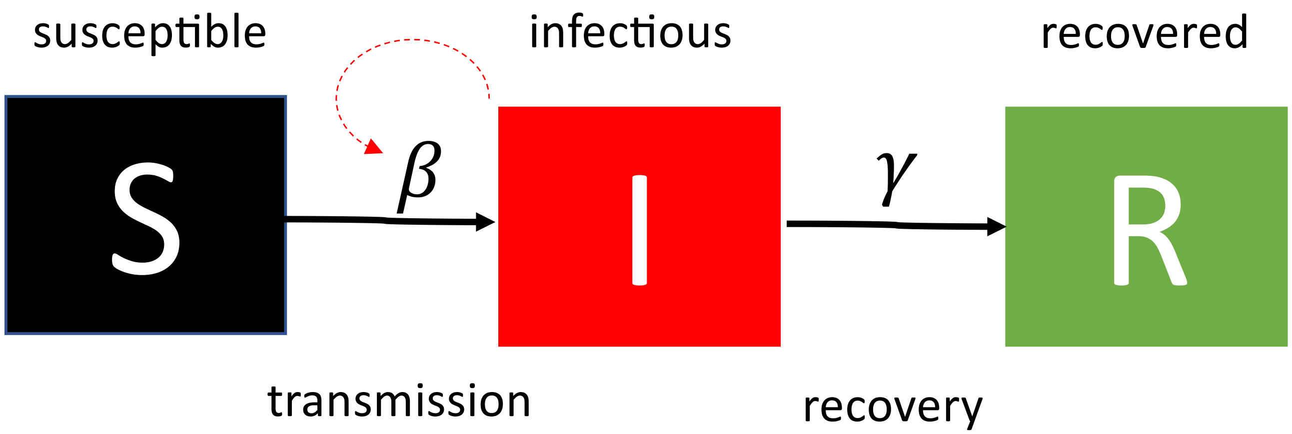
The basic population model





	Discrete time	Continuous time
Rate of change	$\lambda = N_{t+1}/N_t$	$r = [N(t) - N(0)]/t$
Change equation → Can be solved iteratively	$N_{t+1} = \lambda N_t$	$dN(t)/dt = rN(t)$
State equation → Can plug in t to solve for N(t)	$N_t = \lambda^t N_0$	$N(t) = N(0)e^{rt}$

How do we use this model to calculate S(t), I(t), and R(t)?

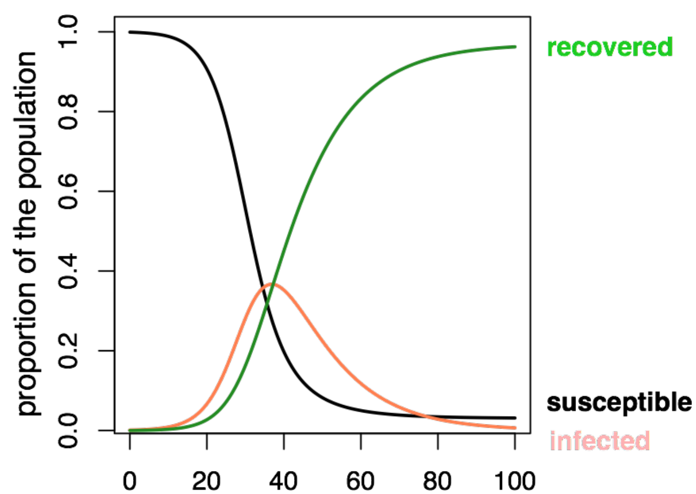


Change equation

$$\frac{dS}{dt} = -\beta S I$$
$$\frac{dI}{dt} = \beta S I - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

State equation

$$S(t) = ?$$
$$I(t) = ?$$
$$R(t) = ?$$



Euler’s method

Imagine that you are driving a car down a highway.

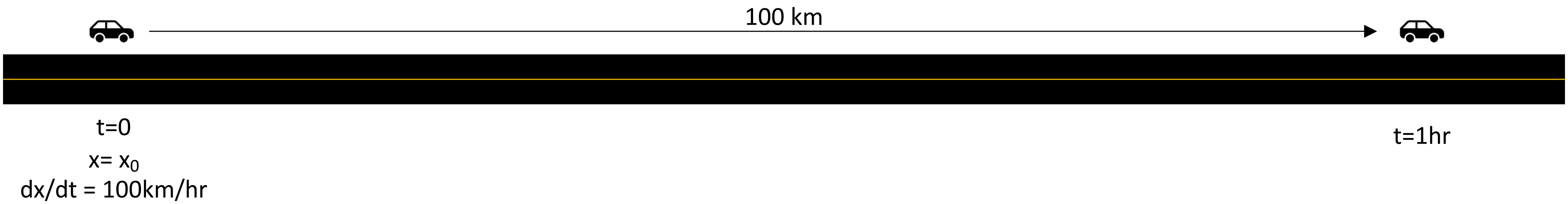
At time t=0, we measure the car’s speed is 100km/hr.

t=0
dx/dt = 100km/hr

Euler's method

Imagine that you are driving a car down a highway.

At time t=0, we measure the car's speed is 100km/hr.



We can estimate the car's position:

1 hr later $x = x_0 + 100\text{km/hr}*1\text{hr} = x_0 + 100\text{km}$

Euler's method

Imagine that you are driving a car down a highway.

At time t=0, we measure the car's speed is 100km/hr.

We can estimate the car's position:

1 hr later $x = x_0 + 100\frac{km}{hr} * 1hr = x_0 + 100km$

1s later $x = x_0 + 100\frac{km}{hr} * \frac{1}{3600}hr = x_0 + 0.278km$

Euler's method

Imagine that you are driving a car down a highway.

At time t=0, we measure the car's speed is 100km/hr.

🤔 Which estimate do you trust most?

We can estimate the car's position:

1 hr later $\quad x = x_0 + 100\frac{km}{hr} * 1hr \quad = x_0 + 100km$

1s later $\quad x = x_0 + 100\frac{km}{hr} * \frac{1}{3600}hr \quad = x_0 + 0.278km$

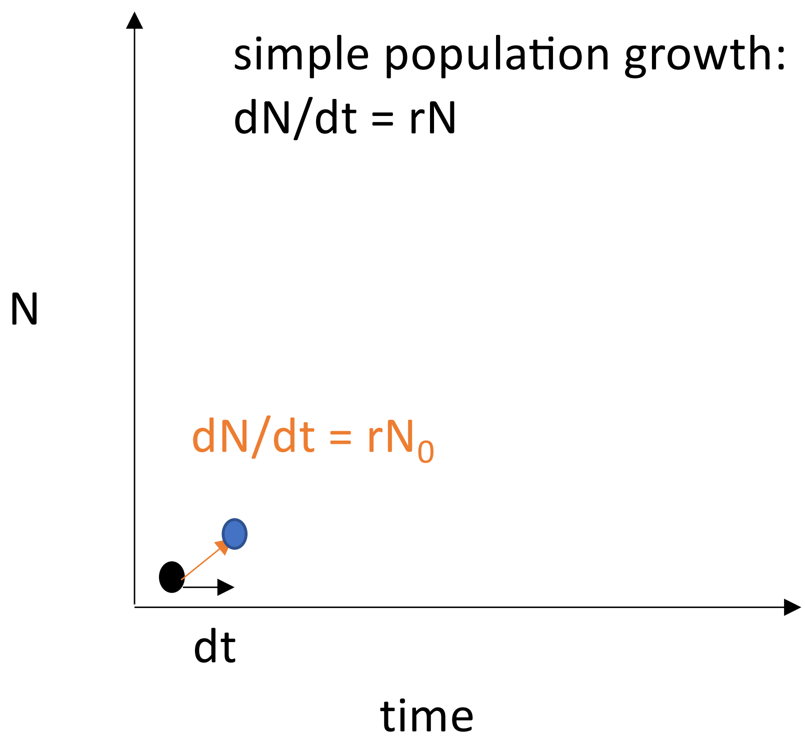
Euler's method

We can use Euler's method to estimate changes in population size:

1. Evaluate dN/dt
2. Calculate the state of the system after dt units of time elapse:

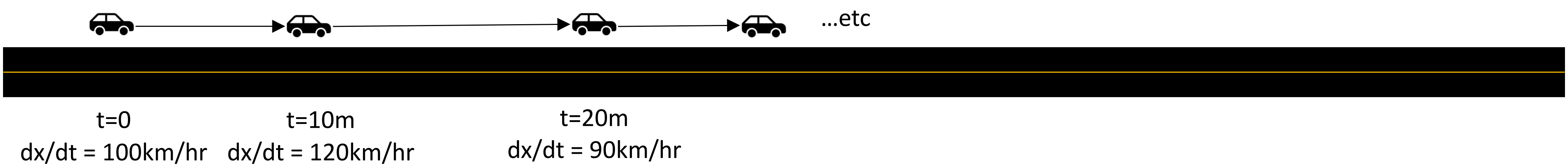
   N(t+dt) = N(t) + dN/dt * dt

next state = current state + change between t and dt



Euler's method

If we needed to estimate the car's position accurately after 1h, we could use Euler's method:



Repeat the following steps until you reach 1h.

1. Measure the car's speed.
2. Project forward dt time units to estimate the car's position at the next time step.

Mathematically:

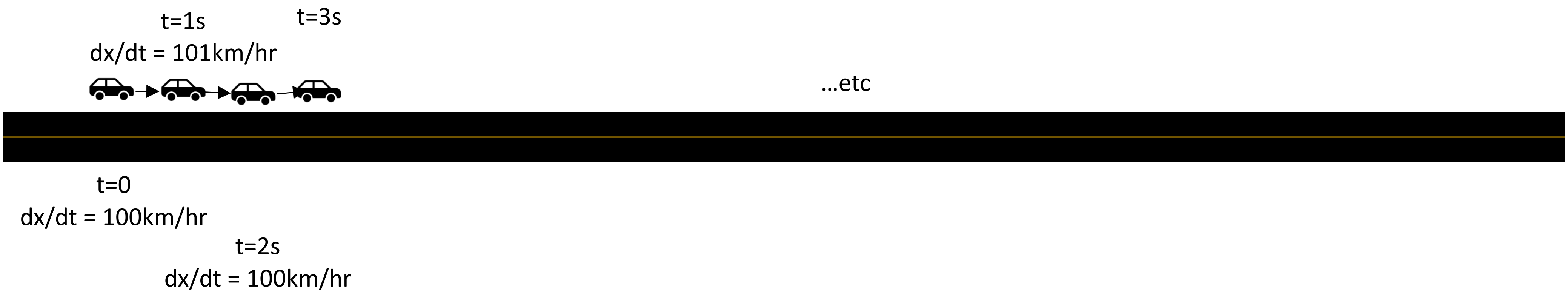
1. Evaluate dx/dt
2. Calculate the state of the system after dt units of time elapse:

   x(t+dt) = x(t) + dx/dt * dt

next state = current state + change between t and dt

Euler's method

If we needed to estimate the car's position accurately after 1h, we could use Euler's method:



Repeat the following steps until you reach 1h.

1. Measure the car's speed. (evaluate dx/dt)
2. Project forward dt time units to estimate the car's position at the next time step.

Smaller dt:

More accurate estimates

More computation

Euler's method

We can use Euler's method to estimate changes in population size:

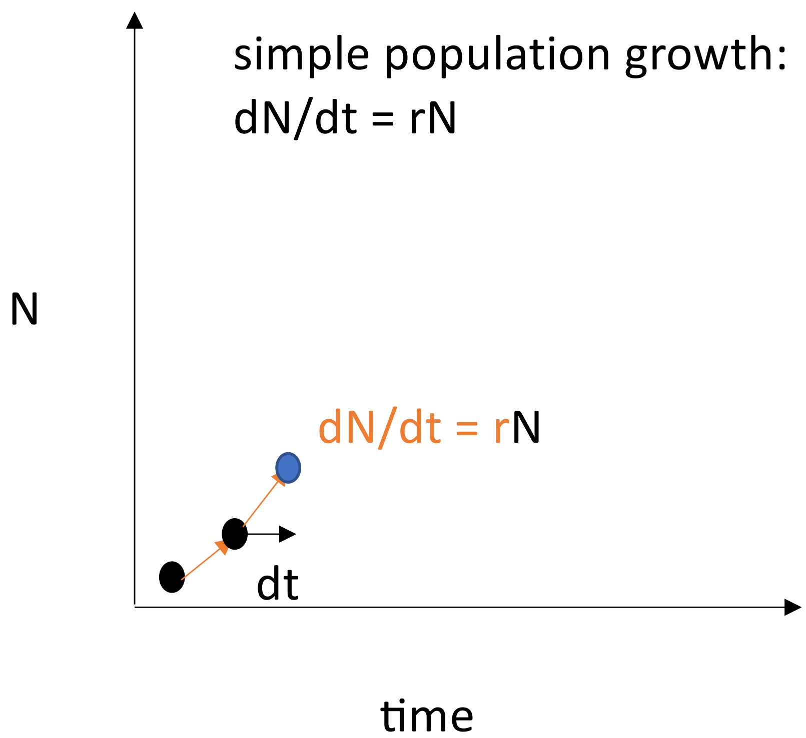
1. Evaluate dN/dt
2. Calculate the state of the system after dt units of time elapse:

   $N(t+dt) = N(t) + dN/dt * dt$

next state = current state + change between t and dt

simple population growth:

$dN/dt = rN$



Euler’s method

We can use Euler’s method to estimate changes in population size:

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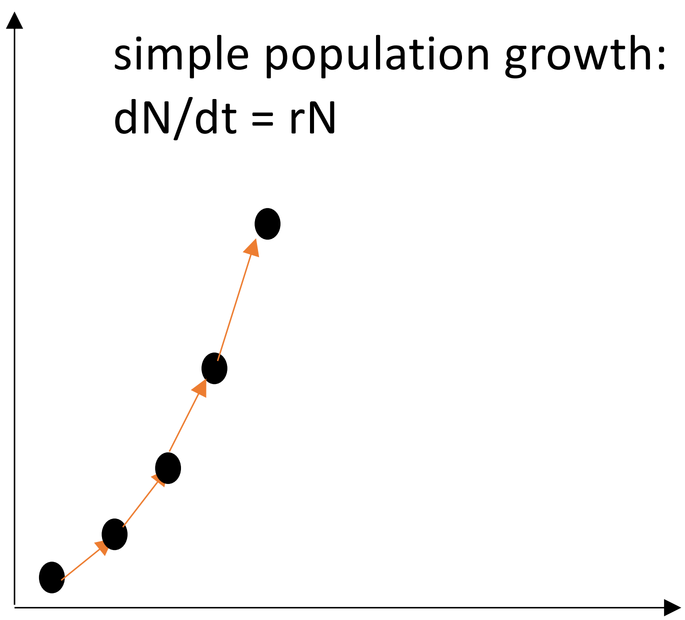

N

time

Practical problems

Euler's method approximates the dynamics of the system, but it is not an exact solution.

Speed/accuracy trade-off: We need to choose a dt small enough to be accurate, but large enough to be computationally feasible.

To avoid these problems, we almost always use R's built-in ODE solver, lsoda().

The computer uses a method similar to Euler integration.
But the computer chooses the step size for us, in a way that maximizes speed while guaranteeing a high level of accuracy.

Key concepts

- We cannot usually solve to find a state equation to simulate the dyanmics of a biological system from a compartmental model.
- Instead, we use an ODE solver in R.
- We could also use Euler's method (discrete approximation). But the accuracy of our solution is sensitive to the size of the timestep, dt. There is a practical tradeoff between speed (larger timesteps) and accuracy (smaller timesteps).

How are we doing?

Good! 🙂 ✅ ❤️ 👍

Bad! 👎 😮 ❌ 🤯

(or ask a question in the chat)

Tutorial

1. Simulate the continuous population growth model three ways:
   a. Using the state equation, N(t) = N0*exp(rt)
   b. Using Euler's method (discrete approximation)
   c. Using an ODE solver
2. Simulate the dynamics of a predator-prey system using an ODE solver
3. Simulate the dynamics of an SIR model using an ODE model
4. Modify the SIR model to an SEIR model