Introduction to Compartmental Models

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Adapted from slides by:
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Amy Wesolowski, Johns Hopkins University
Jessica Metcalf, Princeton University

How are we doing?

Good! 🙂 ✅ ❤️ 👍

Bad! 👎 😮 ❌ 🤯
(or ask a question in the chat)

Question:

What is your favorite animal?

Type answers into the chat.

Goals for this lecture

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- Understand the difference between **statistical** and **compartmental models**
- Understand the difference between **parameters** and **state variables**
- Understand the difference between **discrete-time** and **continuous time** models
- Understand how to formalize and conceptualize compartmental models

Schedule

- Today: simple population growth, age structured population growth
- Wendesday: predator prey, SIR models

Compartmental Models

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   Les populations sont subdivisées en compartiments

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3. Compartments and transition rates are determined by biological systems
   Les compartiments et les taux de transition sont déterminés par les systèmes biologiques

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3. Compartments and transition rates are determined by biological systems
   Les compartiments et les taux de transition sont déterminés par les systèmes biologiques

4. Rates of transferring between compartments are expressed mathematically
   Taux de transition entre les compartiments sont exprimés mathématiquement

How are these different from statistical models?

En quoi sont-ils différents des modèles statistiques?

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En quoi sont-ils différents des modèles statistiques?

Most statistical models (e.g. regressions, ANOVAs, t tests, glms) describe static relationships:

> **They describe relationships between independent and dependent variables, without explicitly modeling changes over time.**

La plupart des modèles statistiques (par exemple, régressions, ANOVA, t tests, glms) décrivent des relations statiques:

> **Ils décrivent les relations entre les variables indépendantes et dépendantes, sans modéliser explicitement les changements dans le temps.**

How are these different from statistical models?

En quoi sont-ils différents des modèles statistiques?

Compartmental models are <u>dynamical models</u>:

They track changes over time in a biological system.

They make explicit hypothesis about the biological forces that cause these changes.

Les modèles compartimentés sont des <u>modèles dynamiques</u>:

Ils suivent les changements au fil du temps dans un système biologique.

Ils font des hypothèses explicites sur les forces biologiques qui causent ces changements.

How are these different from statistical models?

En quoi sont-ils différents des modèles statistiques?

Compartmental models are dynamical models:

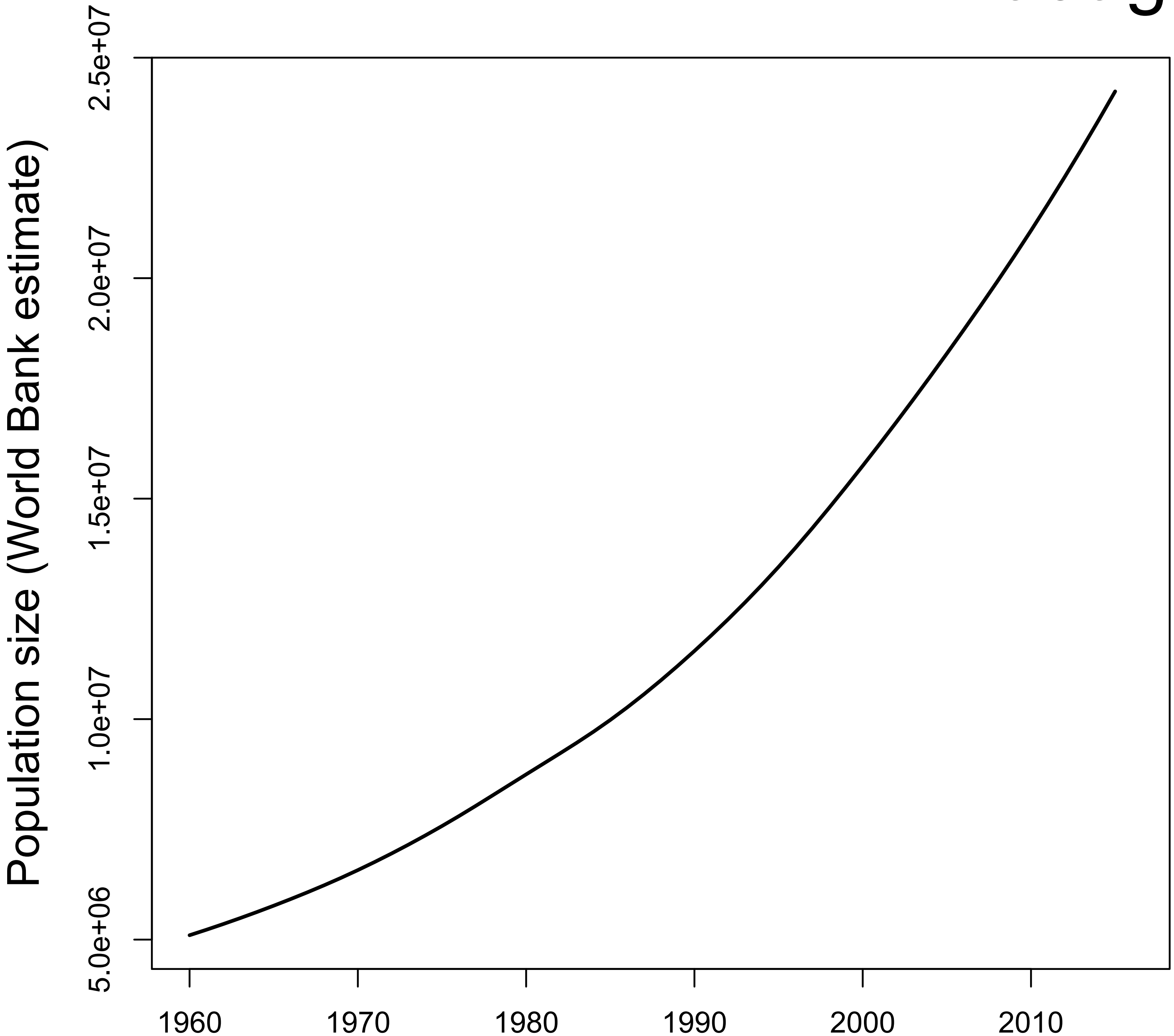
Compartmental models have two main kinds of variables:

1. **State variables – track the number of individuals in each compartment over time**

2. **Parameters – affect rates at which individuals transition between compartments, or interact**

1. Simple Population Models
1. Modèles simples de population

Madagascar



http://databank.worldbank.org

The basic population model

Compartmental models (Mechanistic Models)

1. Populations are divided into compartments
2. Compartments and transition rates are determined by biological systems
3. Rates of transferring between compartments are expressed mathematically
4. Individuals within a compartment are homogenously mix

How does the population of Madagascar grow over time?

Comment est-ce que la population de Madagascar s'augmente avec le passage du temps?

The basic population model

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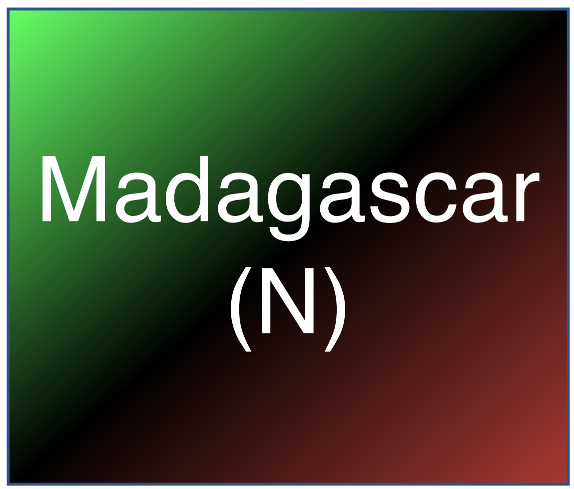

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Madagascar (N)

How does the population grow?

The basic population model

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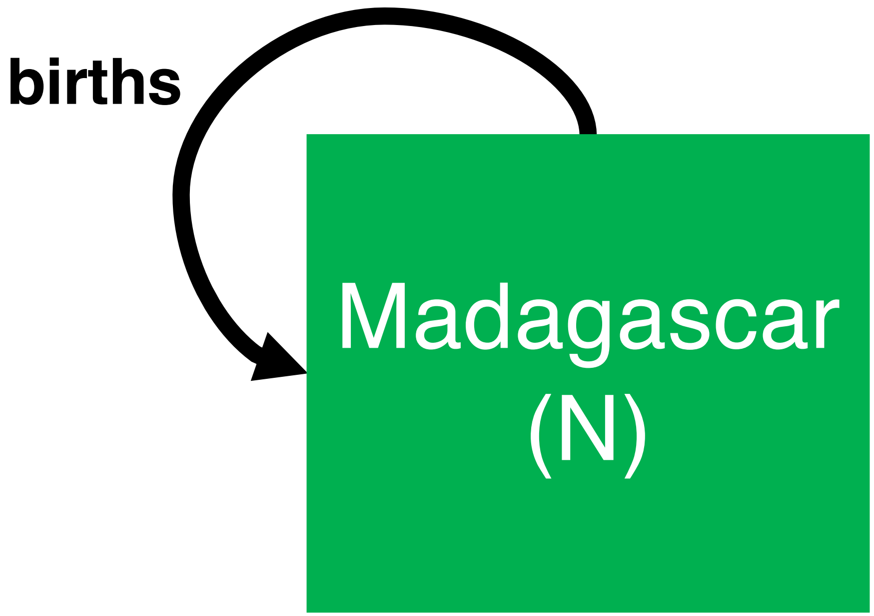

How does the population grow?

The basic population model

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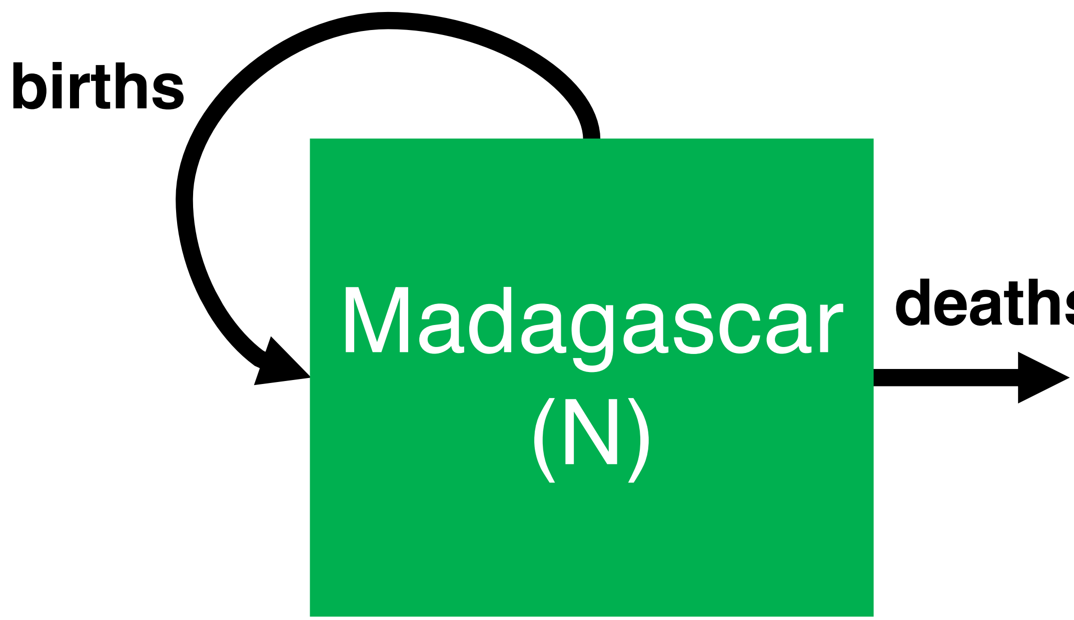

How does the population decrease?

The basic population model

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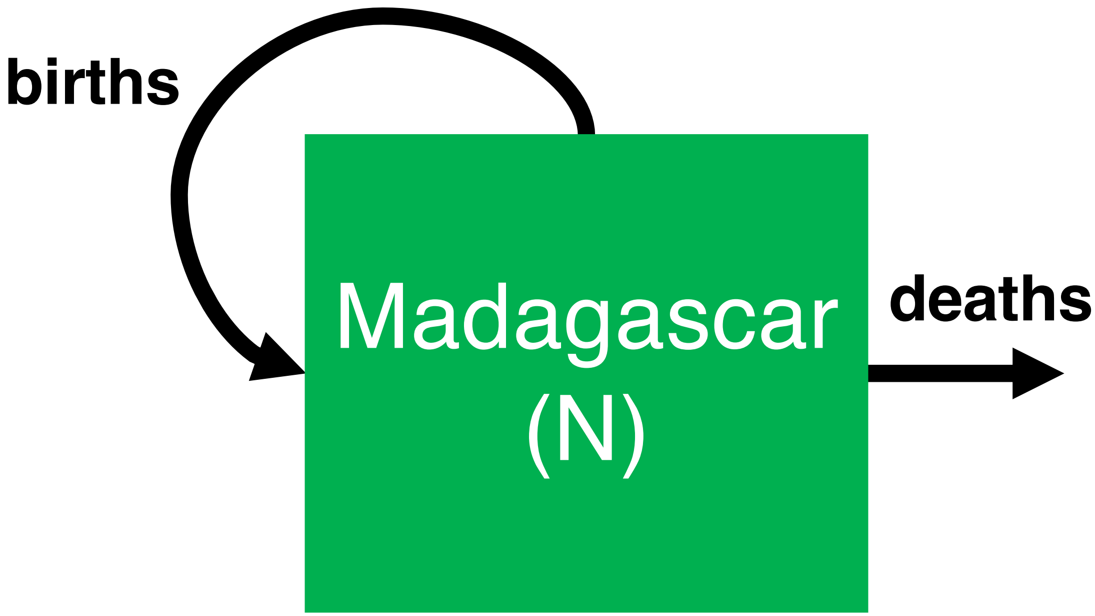

$N_{t+1} =$

The basic population model

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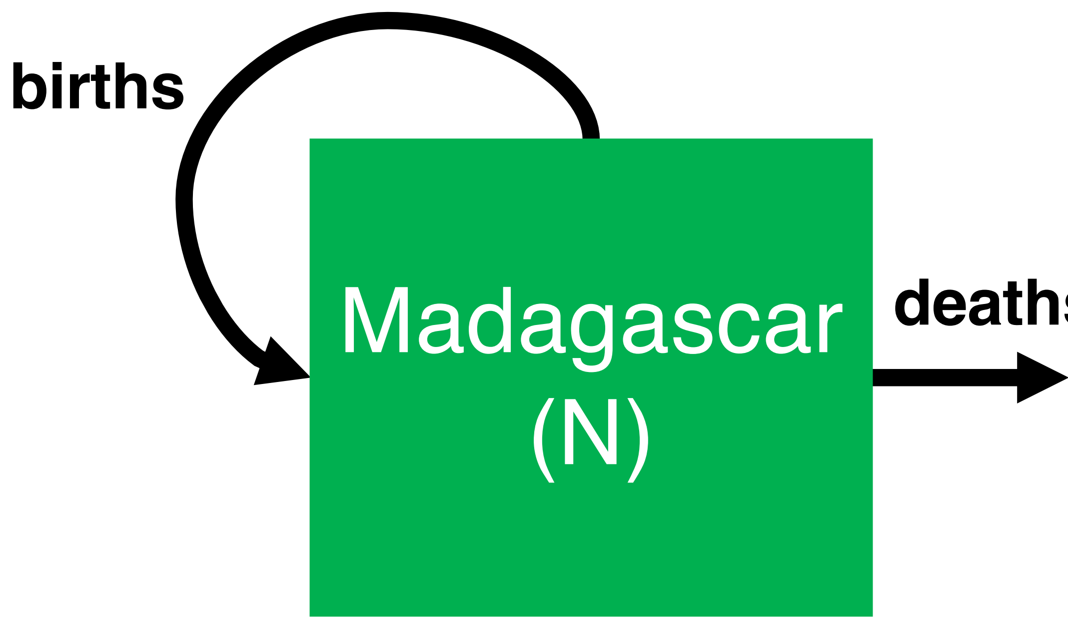

$$N_{t+1}=births*N_t$$

The basic population model

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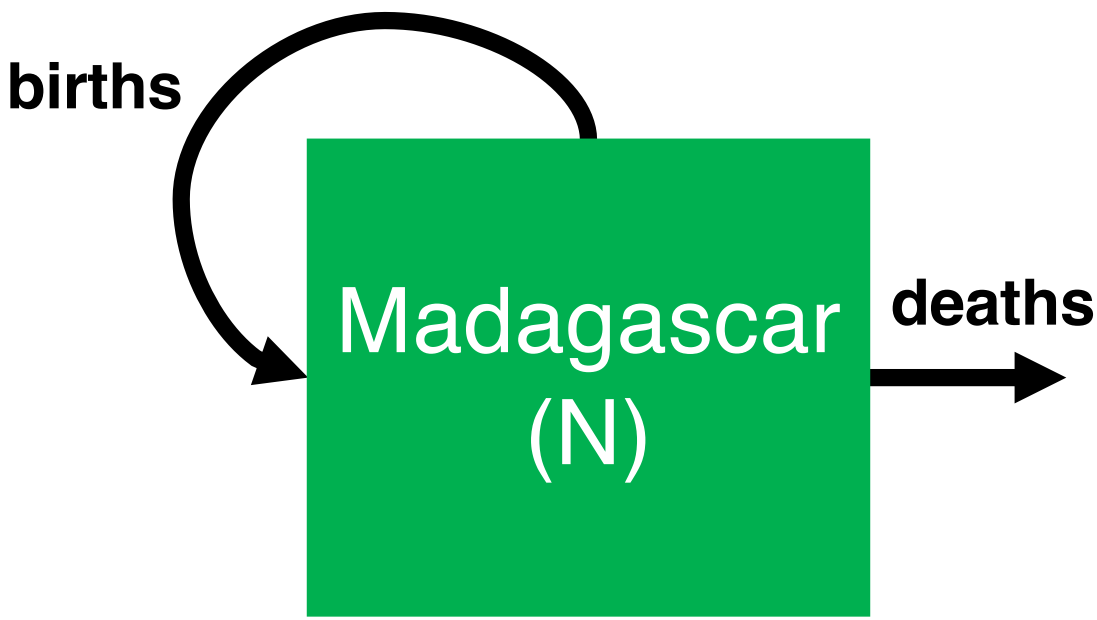

$$N_{t+1}=births*N_t-deaths*N_t$$

The basic population model

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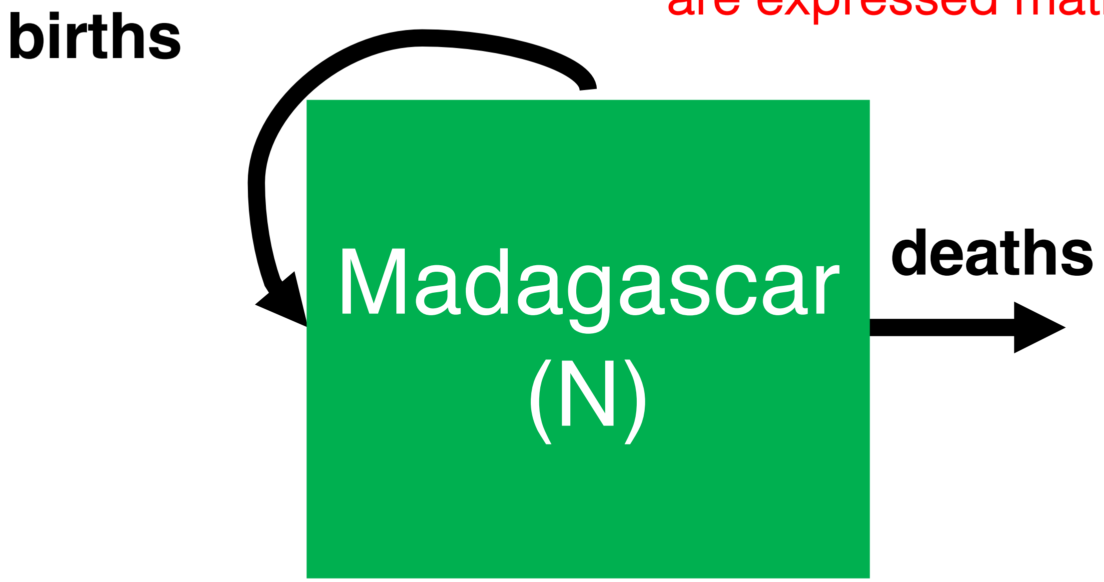


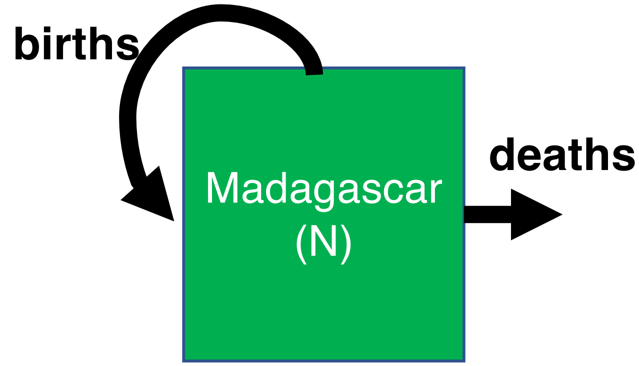
$$N_{t+1}=births*N_t-deaths*N_t$$

$$N_{t+1}=(births-deaths)*N_t$$

$$N_{t+1}=\lambda*N_t$$

λ=pop intrinsic growth rate

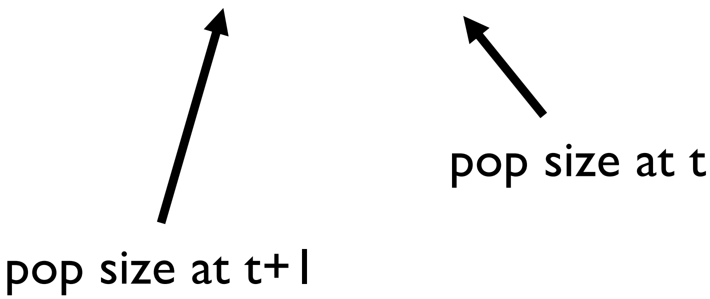
The basic population model



Population rate of increase

Taux d'accroissement de la population

$$\lambda = N_{t+1}/N_t$$

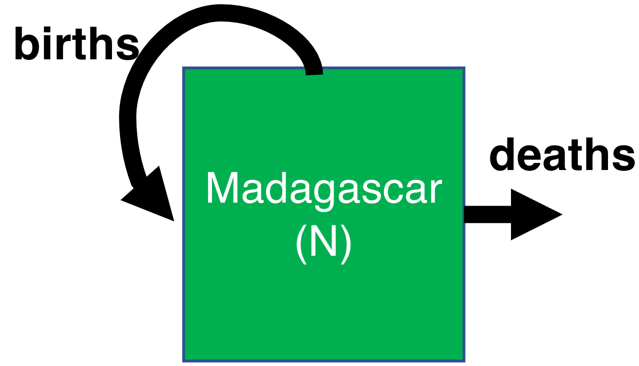


Discrete time

Change equation

$$N_{t+1} = \lambda * N_t$$

The basic population model



Population rate of increase

Taux d'accroissement de la population

$$\lambda = N_{t+1}/N_t$$

pop size at t+1

pop size at t

How are we doing?

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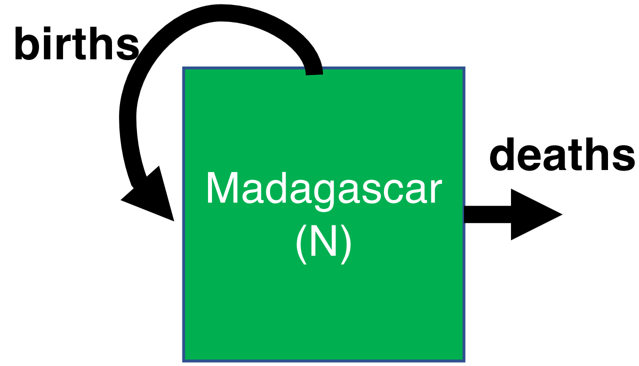
Bad! 👎 😮 ❌ 🤯

(or ask a question in the chat)

Change equation

$$N_{t+1} = \lambda * N_t$$

The basic population model



Population rate of increase

Taux d'accroissement de la population

$$\lambda = N_{t+1}/N_t$$

pop size at t+1

pop size at t

Question:

In this change equation, is λ a <u>parameter</u> or a <u>state variable</u>?

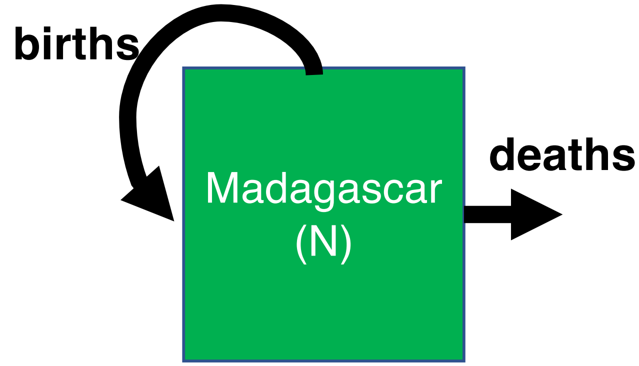
Dans cette équation de changement, λ est-il un paramètre ou une variable d'état ?

Type answers into the chat.

Change equation

$$N_{t+1} = \lambda * N_t$$

The basic population model



Population rate of increase

Taux d'accroissement de la population

$$\lambda = N_{t+1}/N_t$$

pop size at t+1

pop size at t

Change equation

$$N_{t+1} = \lambda * N_t$$

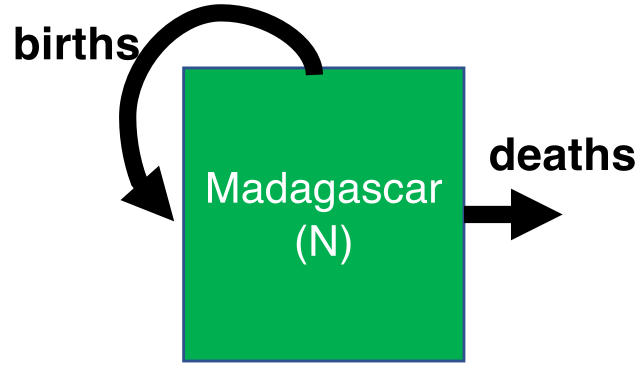
Question:

If λ=0.7, is the population growing or shrinking?

Si λ=0,7, la population augmente-t-elle ou diminue-t-elle ?

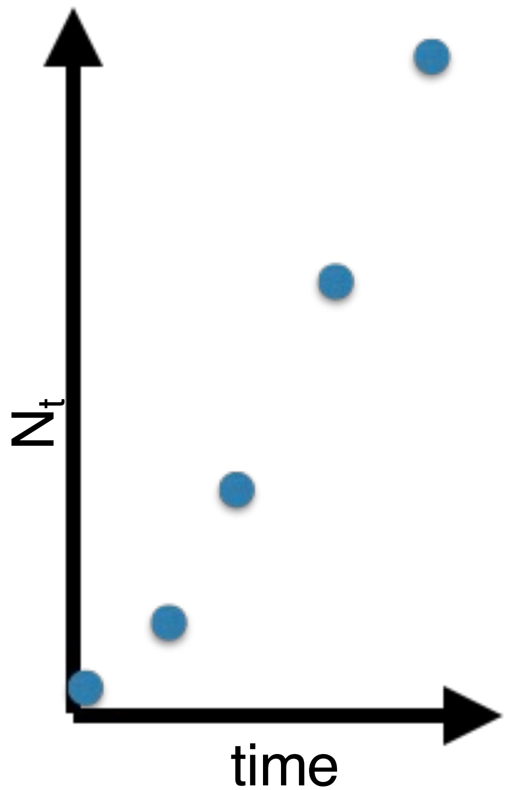
(Type answers into the chat)

The basic population model



Population rate of increase

Taux d'accroissement de la population

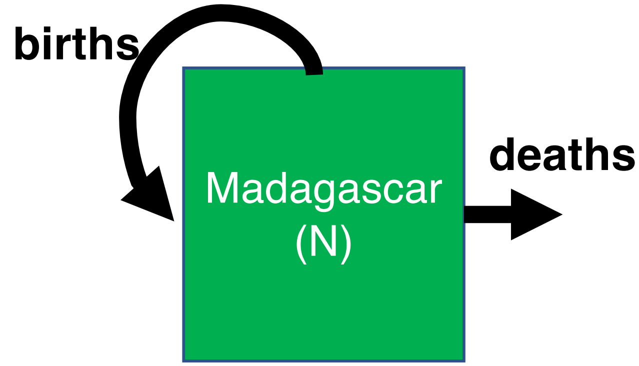


If we start with N_0 individuals at time 0, how many individuals will we have at time 1?

Change equation

$$N_{t+1}=\lambda^{*}N_t$$

The basic population model



Population rate of increase

Taux d'accroissement de la population

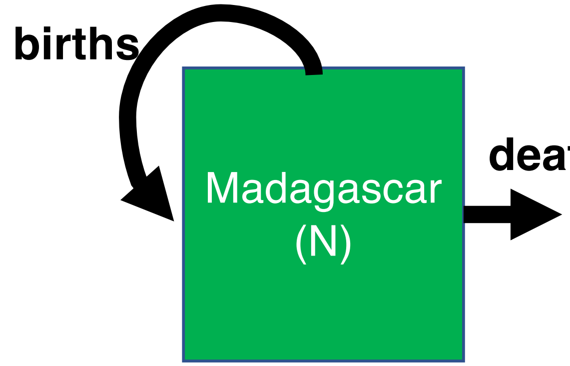
If we start with N_0 individuals at time 0, how many individuals will we have at time 1?

$N_1 = \lambda * N_0$

Change equation

$$N_{t+1} = \lambda * N_t$$

The basic population model



Population rate of increase

Taux d'accroissement de la population

If we start with N_0 individuals at time 0, how many individuals will we have at time 1?

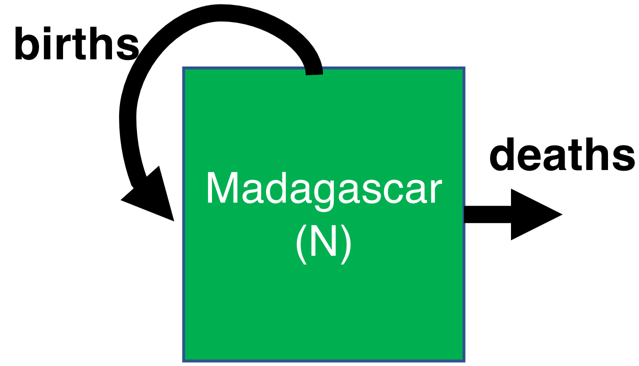
$N_1 = \lambda * N_0$

Then how many individuals will there be at t=2?

Change equation

$$N_{t+1} = \lambda * N_t$$

The basic population model



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$N_1 = \lambda * N_0$

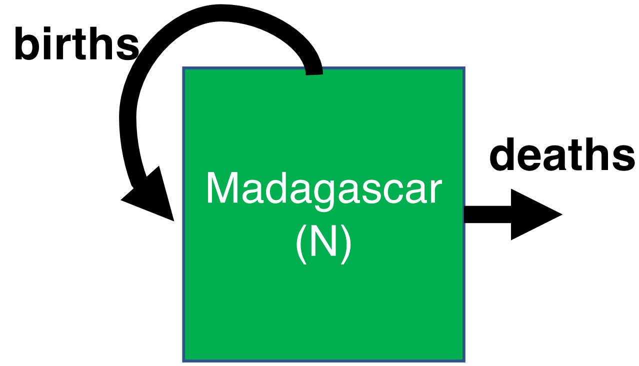
Then how many individuals will there be at t=2?

$N_2 = \lambda * N_1$

Change equation

$$N_{t+1} = \lambda * N_t$$

The basic population model



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Then how many individuals will there be at t=2?

$N_2=\lambda*N_1$

Change equation

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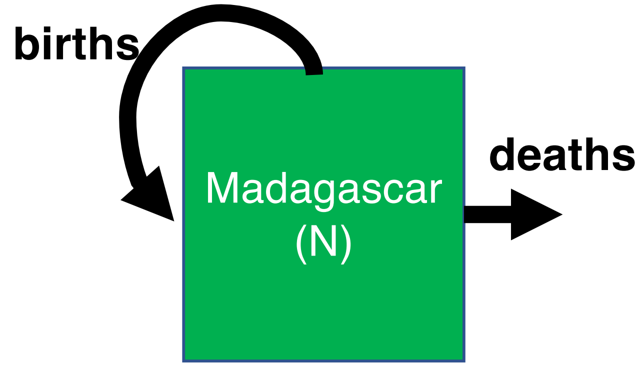
The basic population model



Population rate of increase

Taux d'accroissement de la population

If we start with N_0 individuals at time 0, how many individuals will we have at time 1?

$N_1=\lambda^*N_0$

Then how many individuals will there be at t=2?

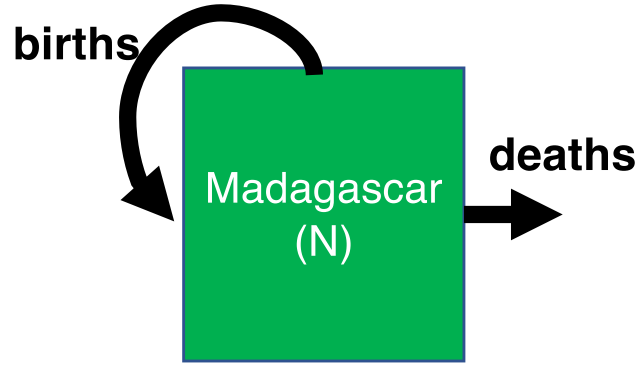
Change equation

$$N_{t+1}=\lambda^*N_t$$

$N_2=\lambda^*N_1$

$N_2=\lambda^*[\lambda^*N_0] = \lambda^2N_0$

The basic population model



Population rate of increase

Taux d'accroissement de la population

If we start with N_0 individuals at time 0, how many individuals will we have at time 1?

$N_1 = \lambda * N_0$

Then how many individuals will there be at t=3?

Change equation

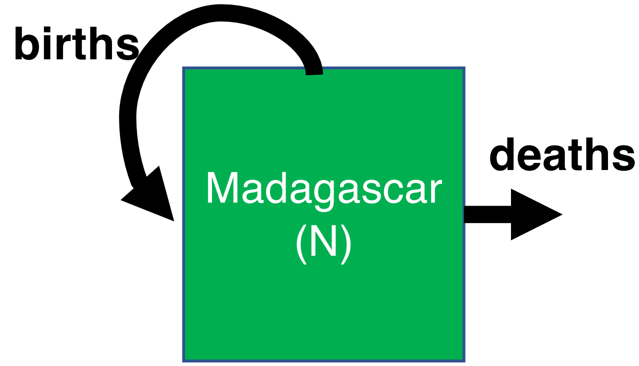
$$N_{t+1} = \lambda * N_t$$

$N_2 = \lambda * N_1 = \lambda^2 N_0$

$N_3 = \lambda * N_2 = \lambda^3 N_0$

...

The basic population model



Population rate of increase

Taux d'accroissement de la population

If we start with N_0 individuals at time 0, how many individuals will we have at time t?

$N_1=\lambda * N_0$

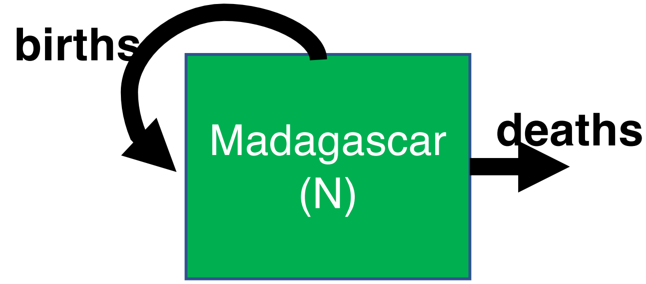
$N_2=\lambda * N_1 = \lambda^2 N_0$

$N_3=\lambda * N_2 = \lambda^3 N_0$

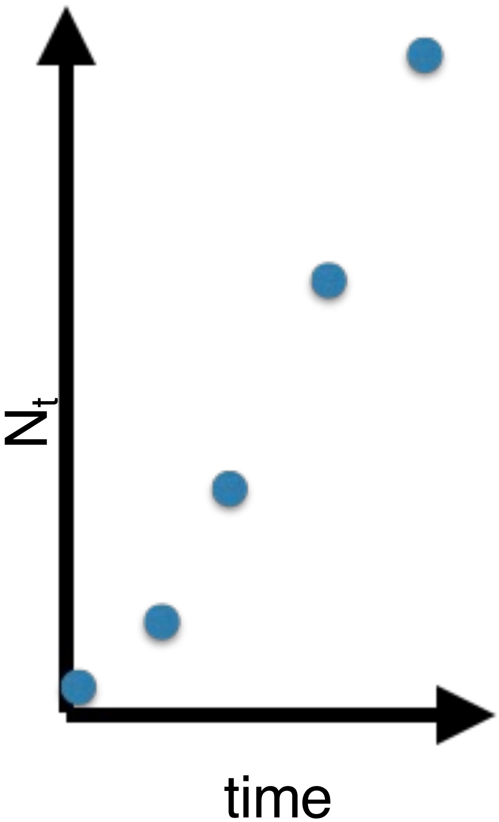
…

$$N_t = \lambda^t N_0$$

Summary: discrete time



Discrete time



Two ways to write down this model:

1. Change equation

→ To find N_t, start from N_0 and solve iteratively

$$N_{t+1}=\lambda * N_t$$

Solve iteratively

$N_1=\lambda * N_0$

$N_2=\lambda * N_1 = \lambda^2 N_0$

$N_3=\lambda * N_2 = \lambda^3 N_0$

...

2. State equation

→ To find N_t, we can plug in t and solve

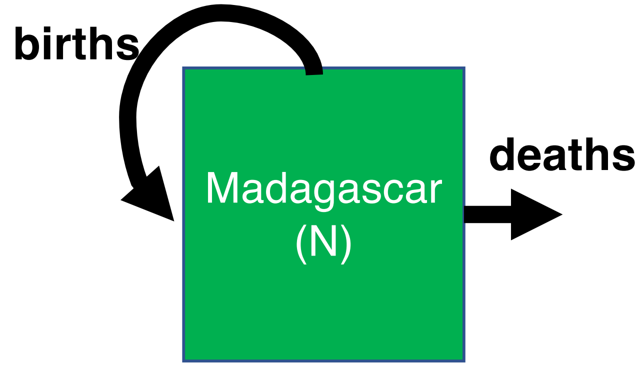
$$N_t = \lambda^t N_0$$

The basic population model

How are we doing?

Good! 🙂 ✅ ❤️ 👍

Bad! 👎 😮 ❌ 🤯
(or ask a question in the chat)



Population rate of increase
Taux d'accroissement de la population

$N_1 = \lambda * N_0$

$N_2 = \lambda * N_1 = \lambda^2 N_0$

$N_3 = \lambda * N_2 = \lambda^3 N_0$

...

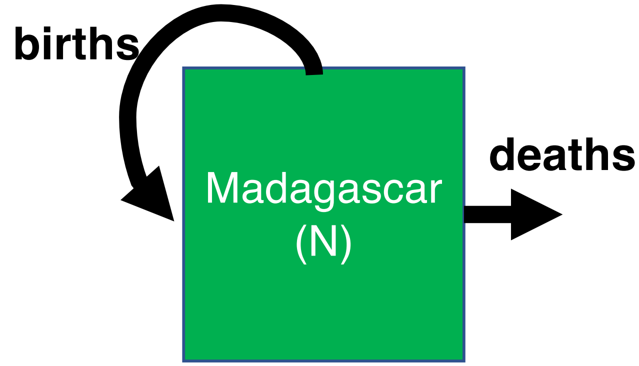
Change equation

$$N_{t+1} = \lambda * N_t$$

State equation

$$N_t = \lambda^t N_0$$

The basic population model



Question:

In what situations would we want a time step NOT equal to one year?

Dans quelles situations souhaiterions-nous un pas de temps NON égal à un an ?

(Type answers into the chat)

Population rate of increase

Taux d'accroissement de la population

$N_1=\lambda^*N_0$

$N_2=\lambda^*N_1 = \lambda^2N_0$

$N_3=\lambda^*N_2 = \lambda^3N_0$

...

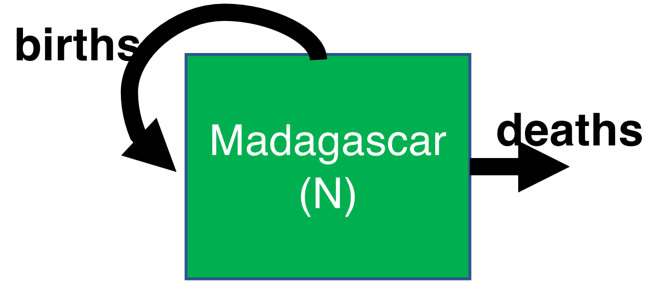
Change equation

$N_{t+1}=\lambda^*N_t$

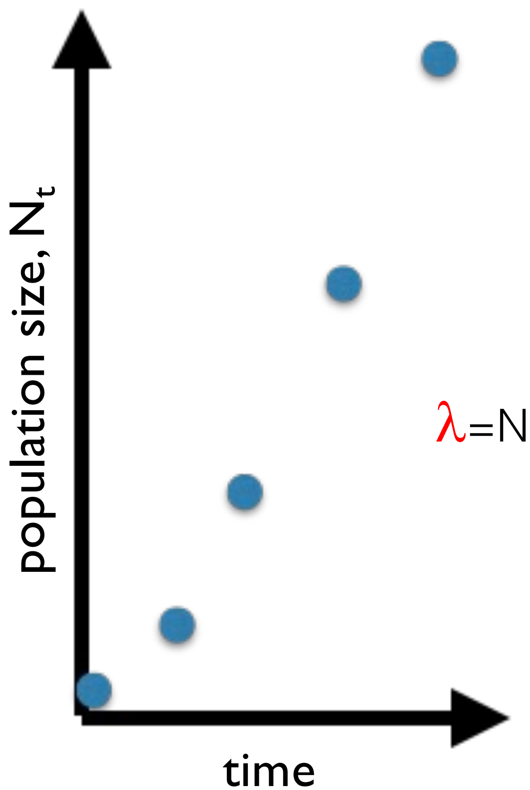
State equation

$N_t = \lambda^tN_0$

The basic population model

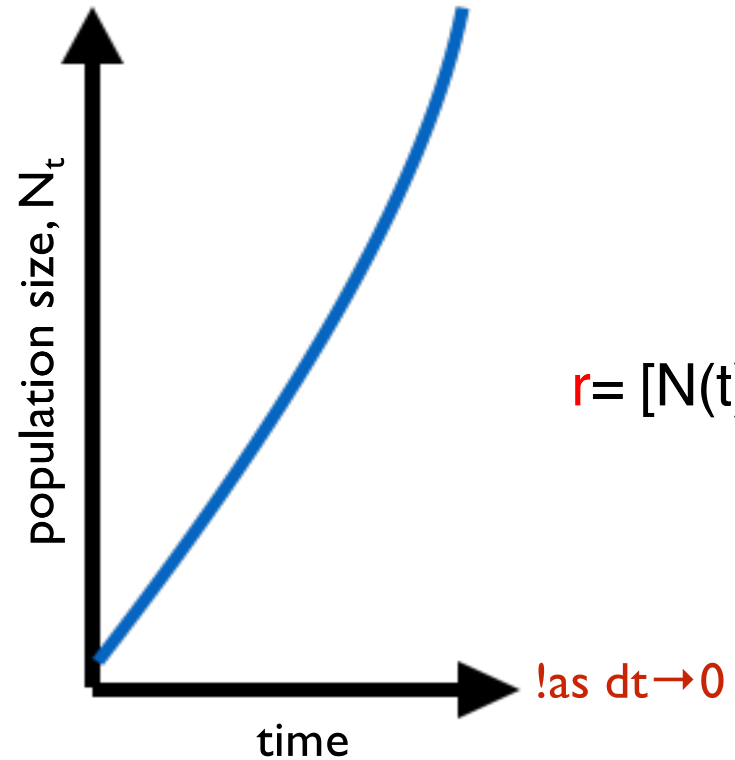


Discrete time



$\lambda = N_{t+1}/N_t$

Continuous time



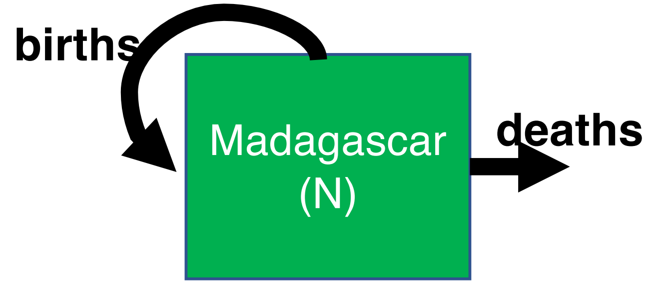
$r = [N(t)-N(0)]/t$

Change equation

$N_{t+1} = \lambda * N_t$

$$\frac{dN(t)}{dt} = rN(t)$$

Solve for the state equation



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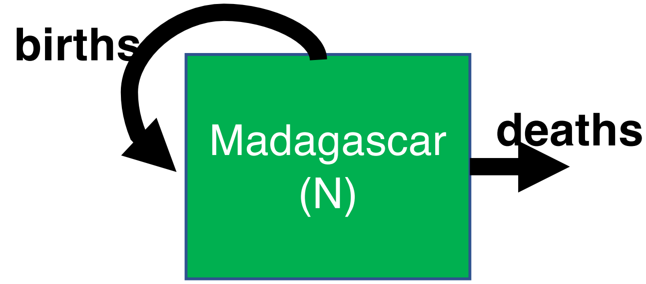


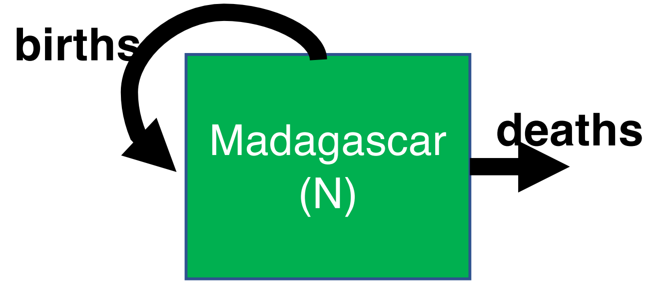
Continuous time

$$\frac{dN(t)}{dt} = rN(t)$$

1. Separation of variables:

$$\frac{dN(t)}{N(t)} = rdt$$

Solve for the state equation



Continuous time

$$\frac{dN(t)}{dt} = rN(t)$$

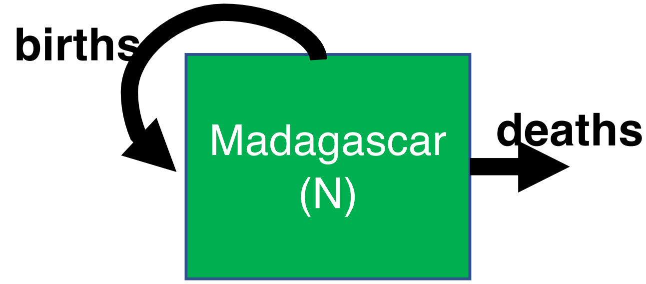
1. Separation of variables:

$$\frac{dN(t)}{N(t)} = r dt$$

2. Integrate both sides:

$$\int \frac{dN(t)}{N(t)} = \int r dt$$

Solve for the state equation



Continuous time

$$\frac{dN(t)}{dt} = rN(t)$$

1. Separation of variables:

$$\frac{dN(t)}{N(t)} = rdt$$

2. Integrate both sides:

$$\int \frac{1}{N(t)} dN(t) = \int rdt$$

By definition:

$$\log(N(t)) + C = rt + C$$

The basic population model

Continuous time

$$\frac{dN(t)}{dt} = rN(t)$$

1. Separation of variables:

$$\frac{dN(t)}{N(t)} = rdt$$

2. Integrate both sides:

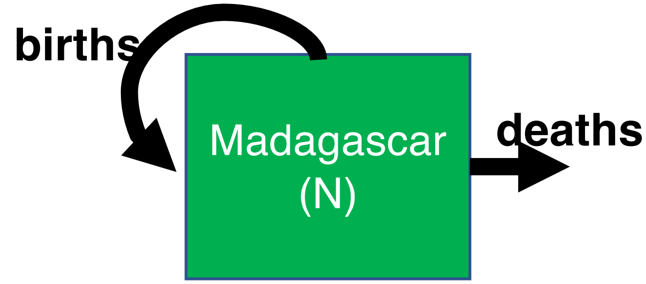
$$\int \frac{1}{N(t)} dN(t) = \int rdt$$

By definition:

$$\log\big(N(t)\big) = rt + \underline{(C_2 - C_1)}$$

the difference of constants is a constant

The basic population model



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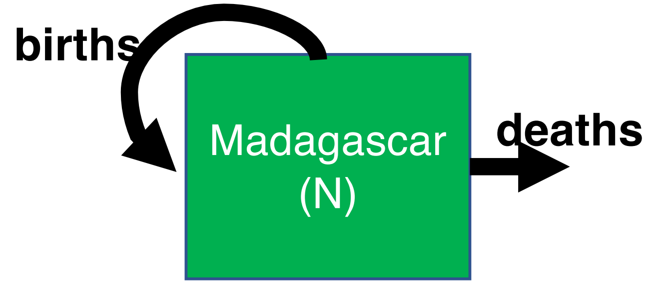

Continuous time

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1. Separation of variables:

$$\frac{dN(t)}{N(t)} = rdt$$

2. Integrate both sides:

$$\int \frac{1}{N(t)} dN(t) = \int rdt$$

By definition:

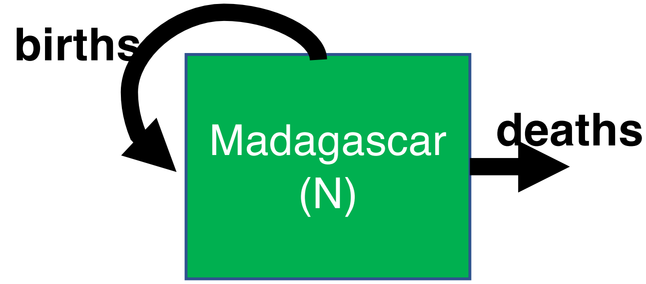
$$\log\big(N(t)\big) = rt + C$$

Take exponentials:

$N(t) = e^{rt + c} = Ce^{rt}$

$N(t) = N_0 e^{rt}$

The basic population model



Continuous time

$$\frac{dN(t)}{dt} = rN(t)$$

1. Separation of variables:

$$\frac{dN(t)}{N(t)} = rdt$$

2. Integrate both sides:

$$\int \frac{1}{N(t)} dN(t) = \int rdt$$

By definition:

$$\log\big(N(t)\big) = rt + C$$

Detailed math

Take exponentials to solve for N(t):

N(t)= $e^{rt+C} = Ce^{rt}$

Substitute [t=0, N=N0] to show that C = N0:

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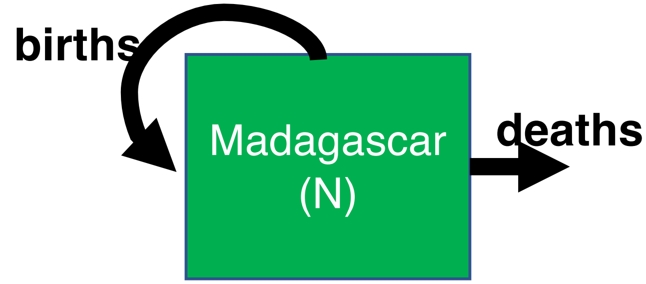

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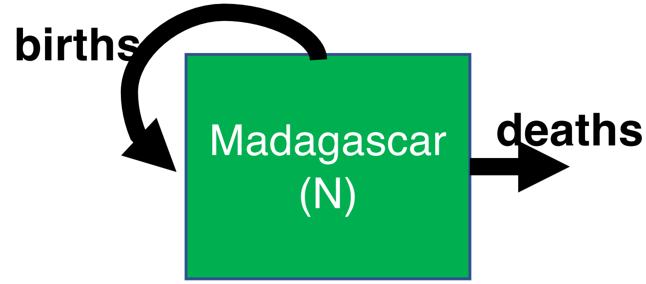

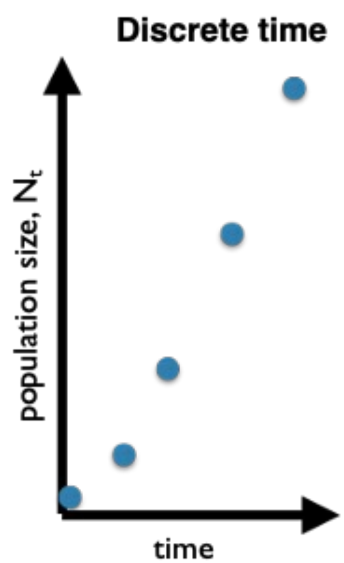

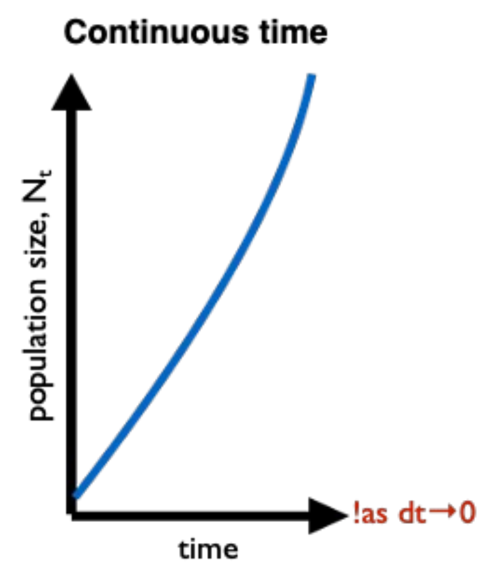

	Discrete time	Continuous time
Rate of change	$\lambda = N_{t+1}/N_t$	$r = [N(t) - N(0)]/t$
Change equation (Can be solved iteratively)	$N_{t+1} = \lambda N_t$	$dN(t)/dt = rN(t)$
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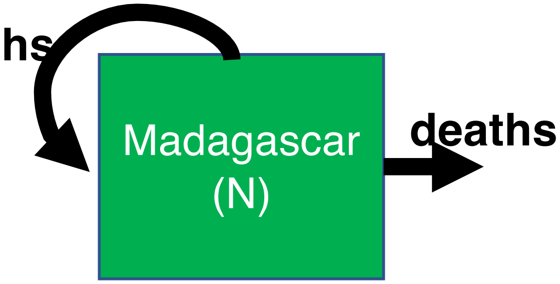
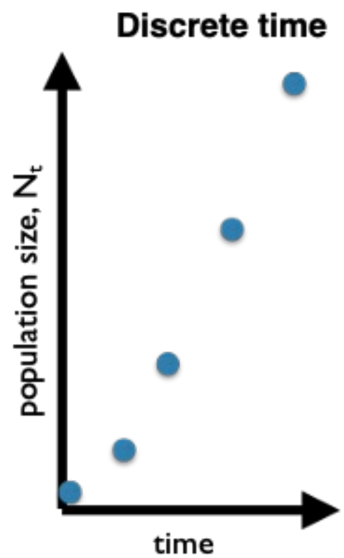
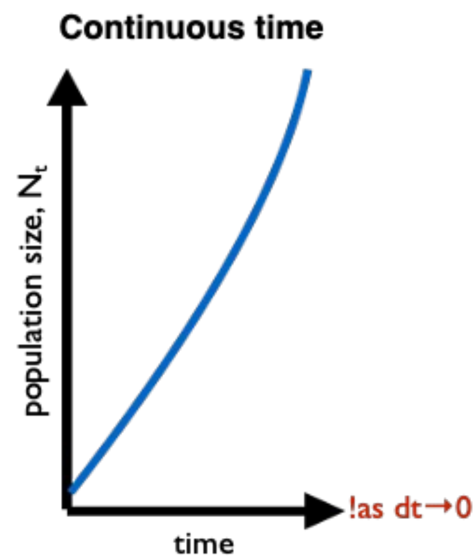
The basic population model



!!
In more complicated models, we cannot always solve for the state equation.
!!





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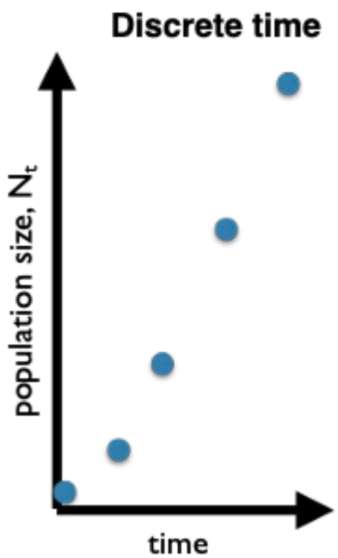
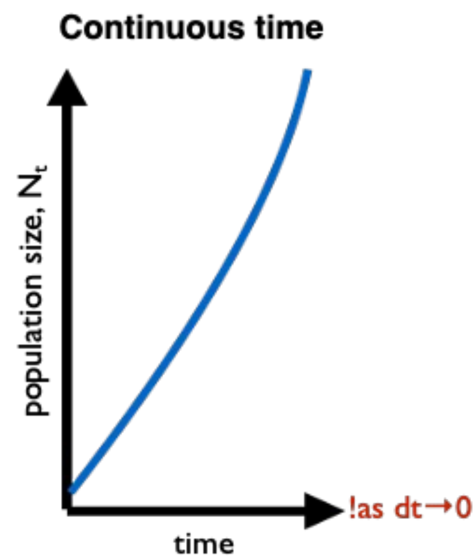
The basic population model

Continuous models can be discretized; discrete models can be approximated by continuous ones. The appropriate framing may depend on the data / question.





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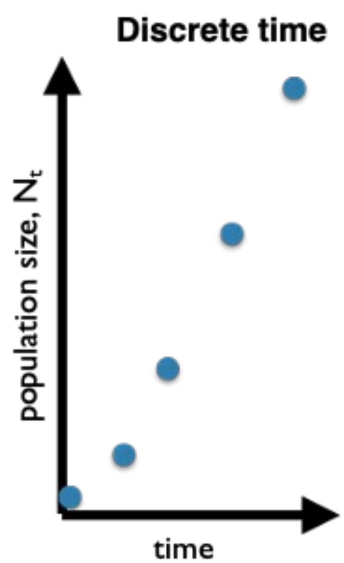
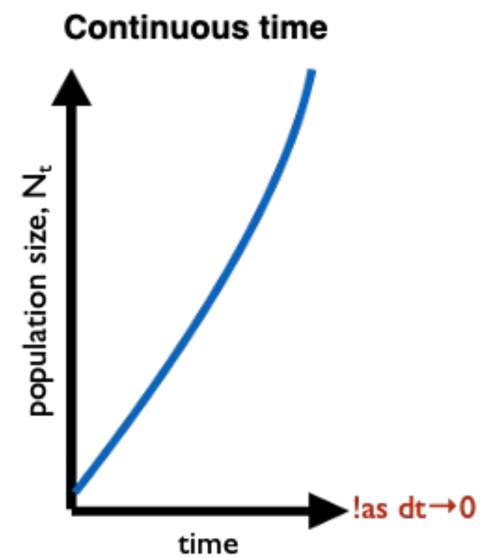
The basic population model

How are we doing?

Good! 🙂 ✅ ❤️ 👍

Bad! 👎 😮 ❌ 🤯
(or ask a question in the chat)





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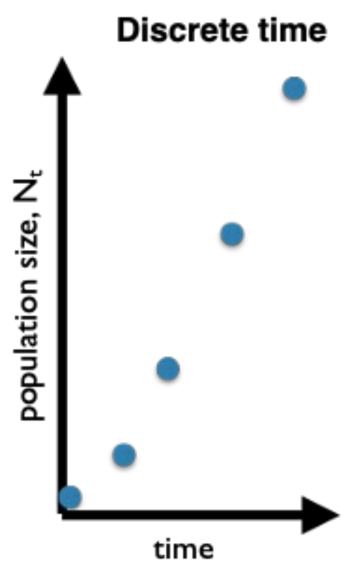
The basic population model

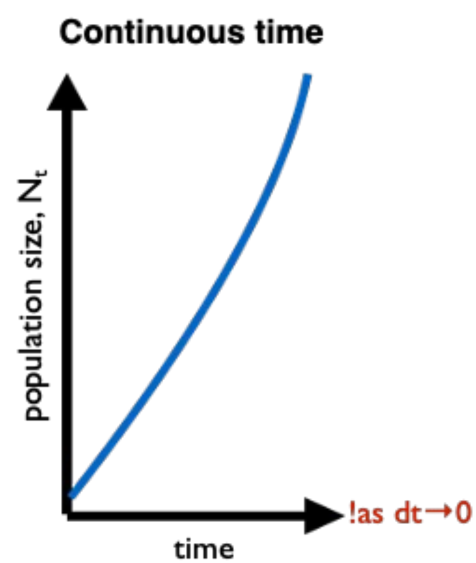
Question:

Exponential growth models assume that the population growth rate, r, is constant. What is the problem with this assumption?

Les modèles de croissance exponentielle supposent que le taux de croissance de la population, r, est constant. Quel est le problème avec cette hypothèse ?

(Answer in the chat)





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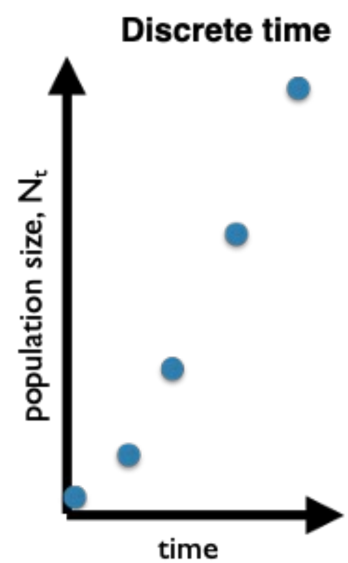
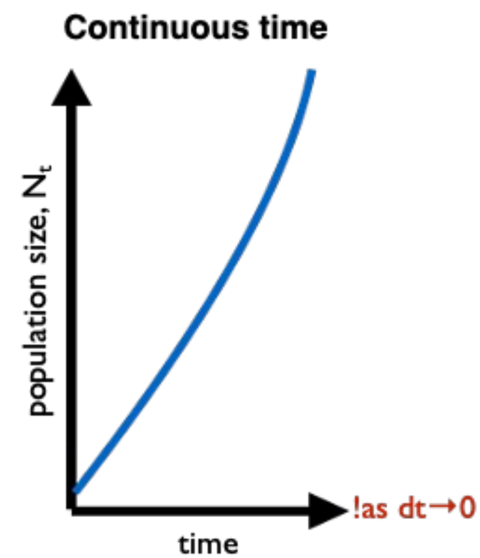
The basic population model

Question:

In what situations is exponential growth a reasonable model?
Dans quelles situations la croissance exponentielle est-elle un modèle raisonnable ?

(Answer in the chat)





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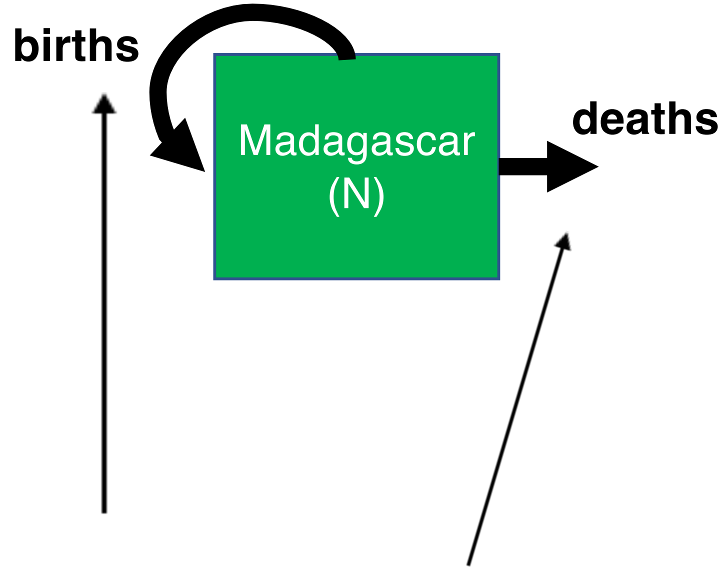
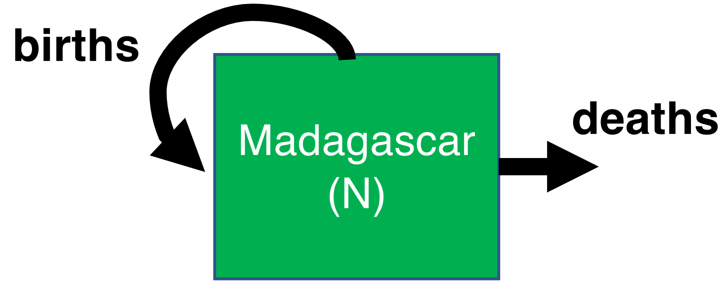
The basic population model

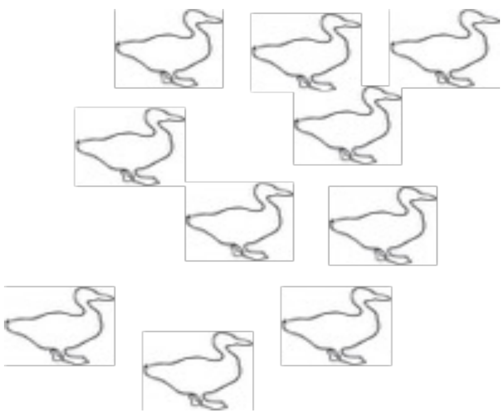


What about those rates?
Are they the same every year?
And in every person?

The basic population model



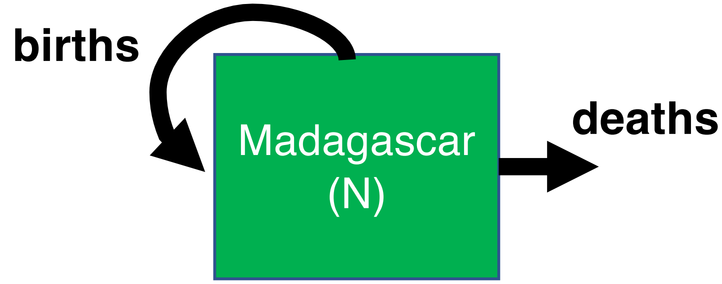
starting population



probability of death = 0.5

if deterministic "always the same"

The basic population model



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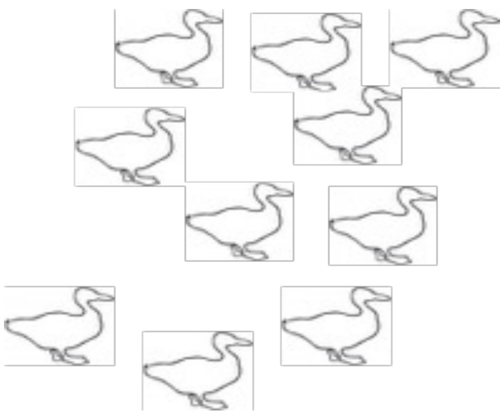

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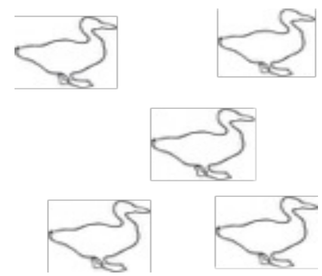

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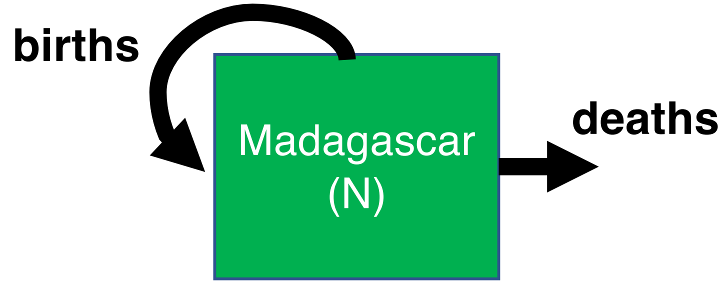

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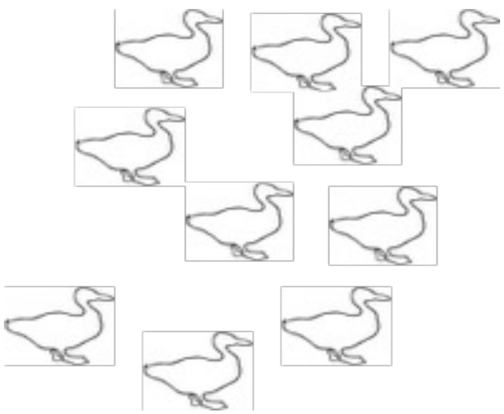

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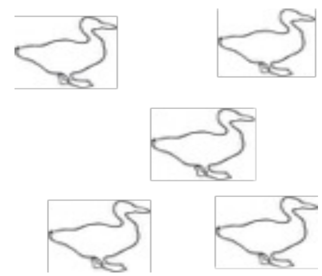

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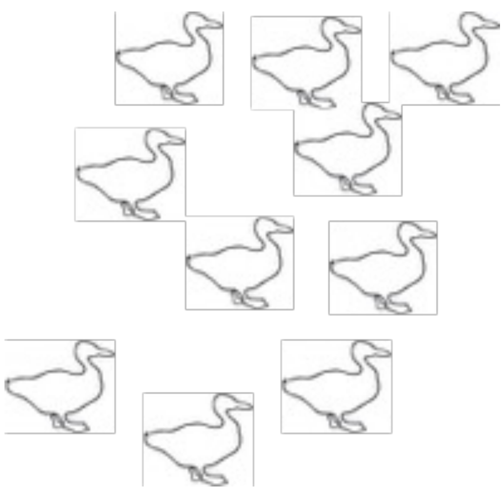

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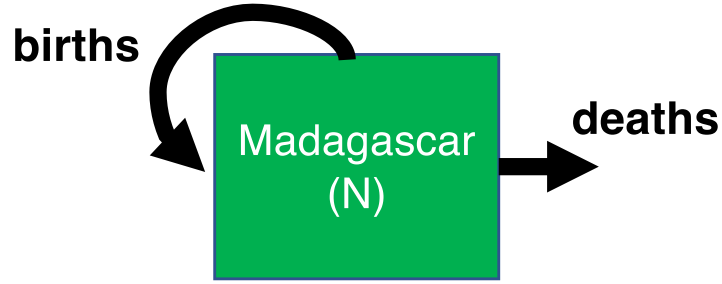
The basic population model



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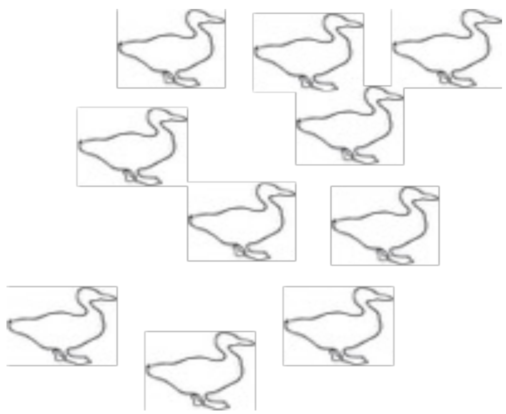

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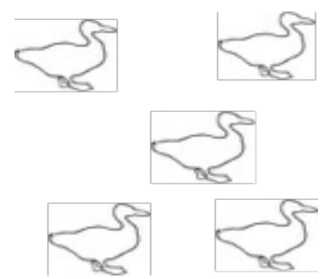

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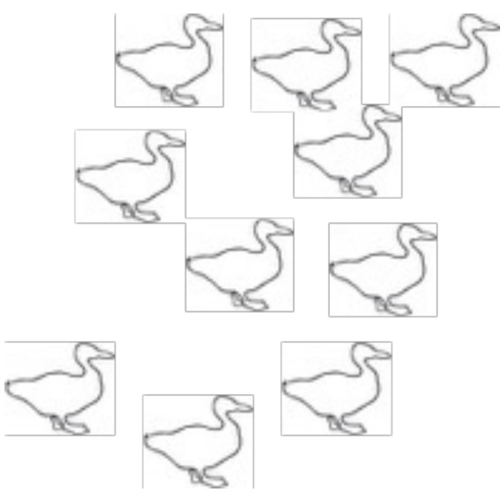

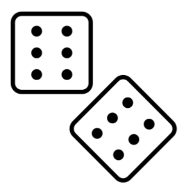

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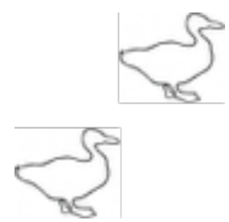
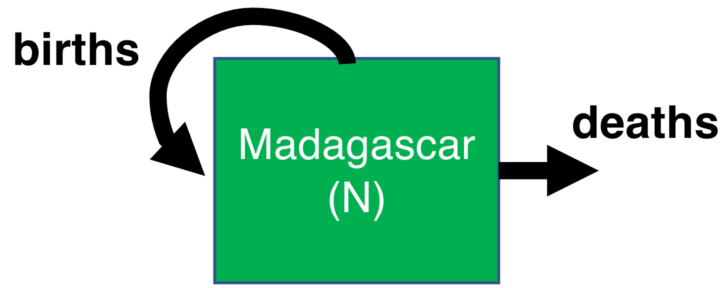
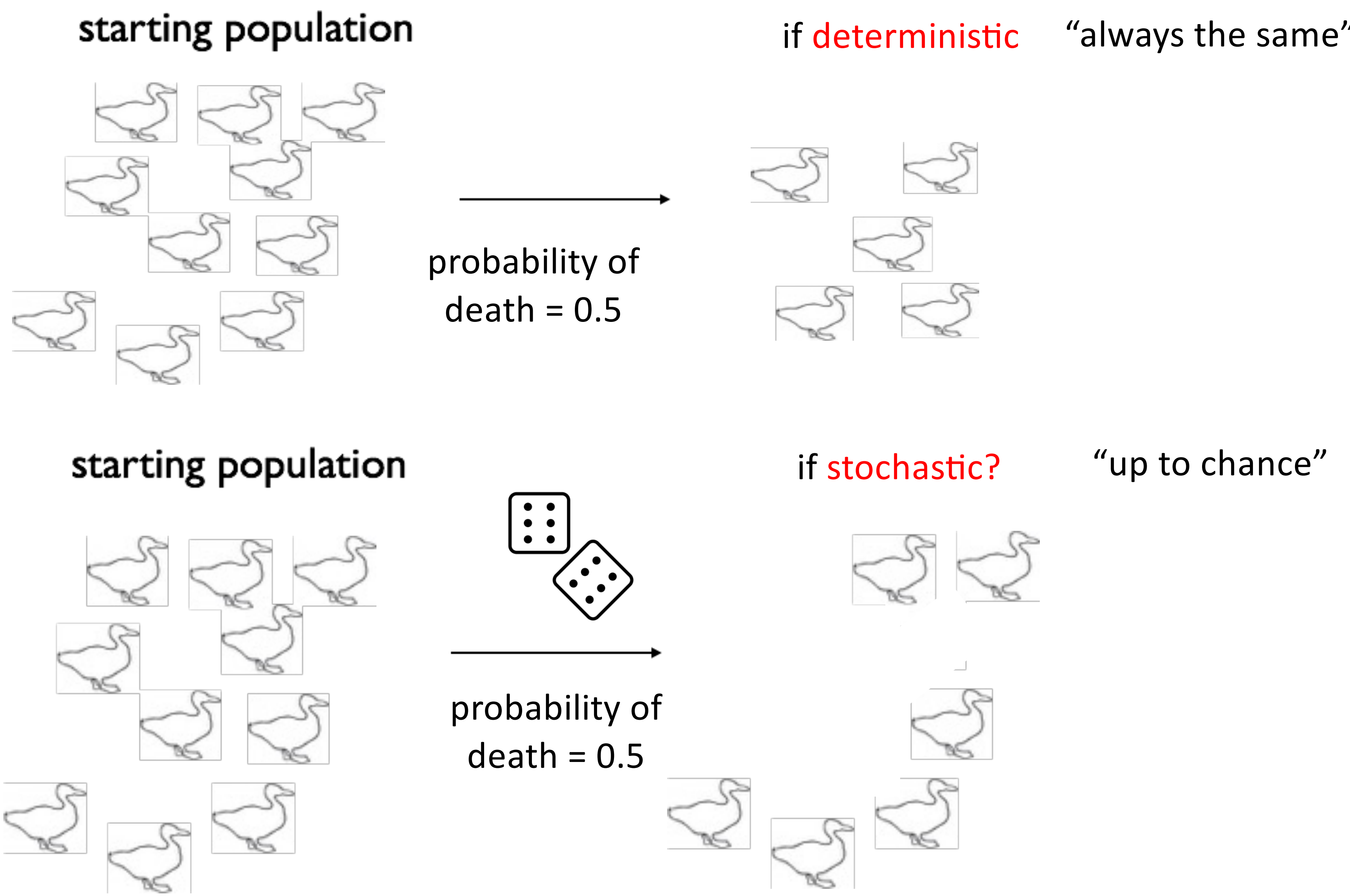


The basic population model





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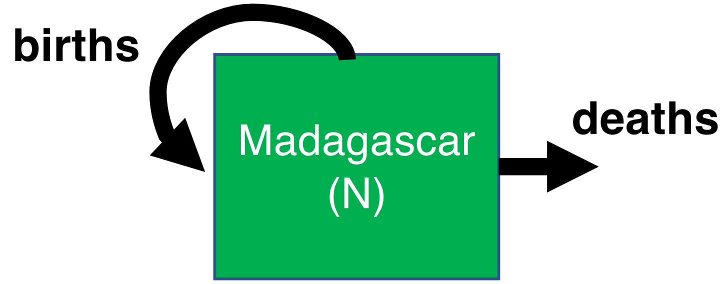

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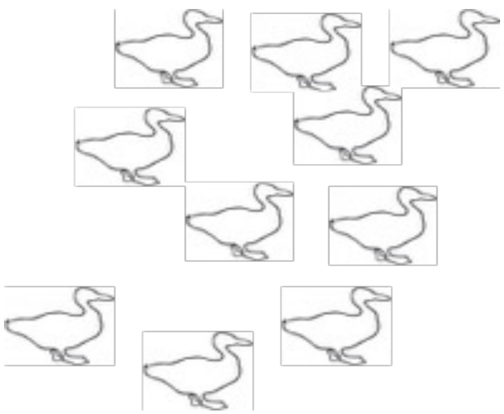

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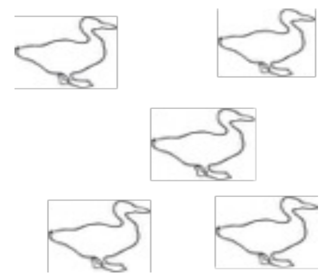

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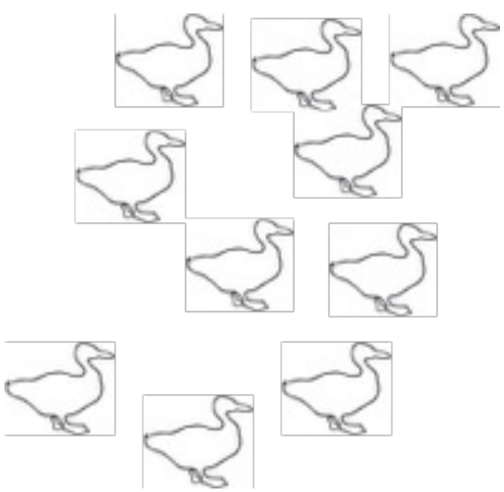

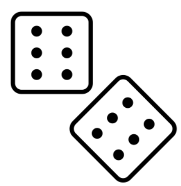

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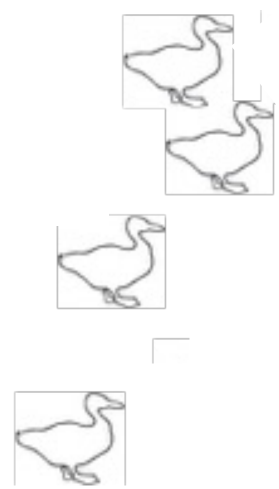

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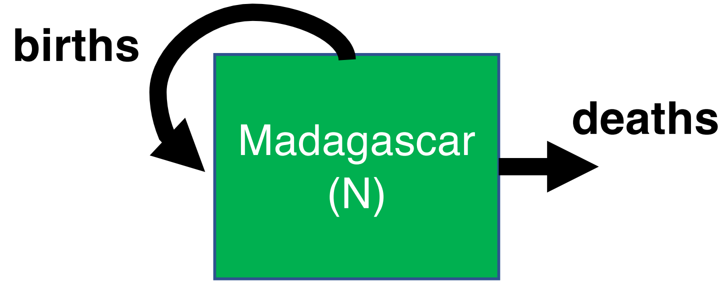

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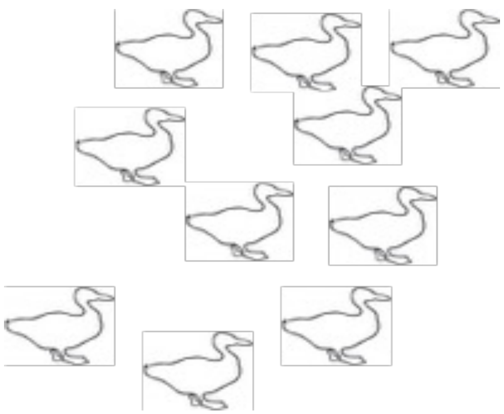

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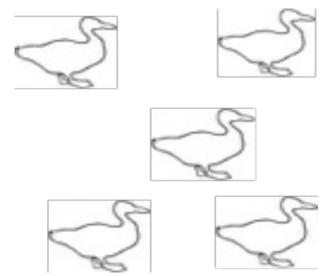
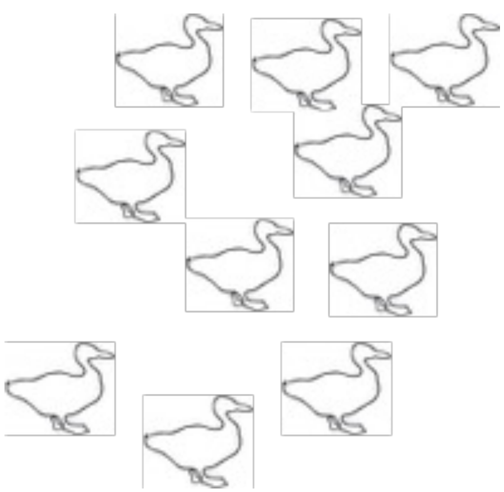


rbinom(200,10,0.5)

starting population



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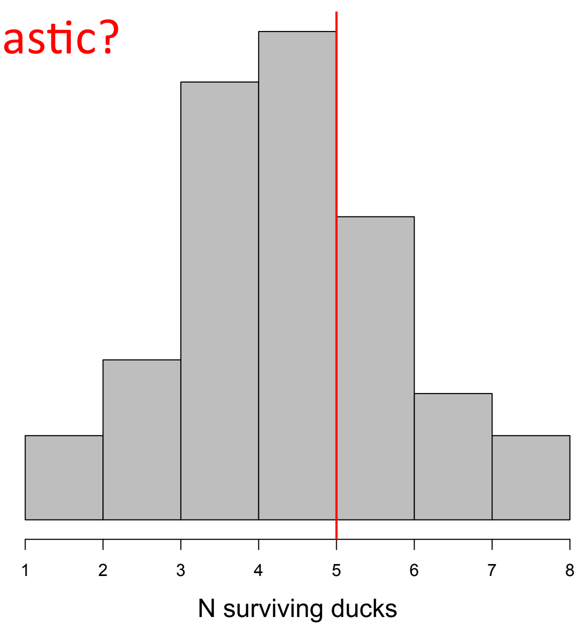
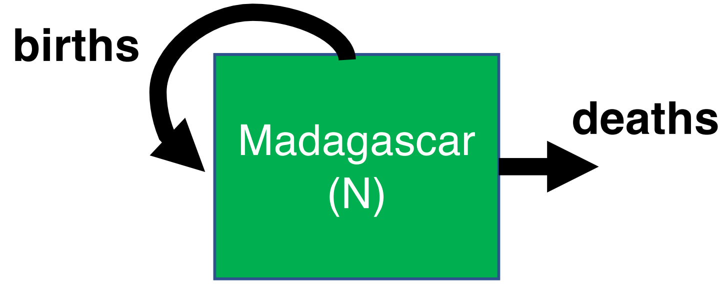


The basic population model



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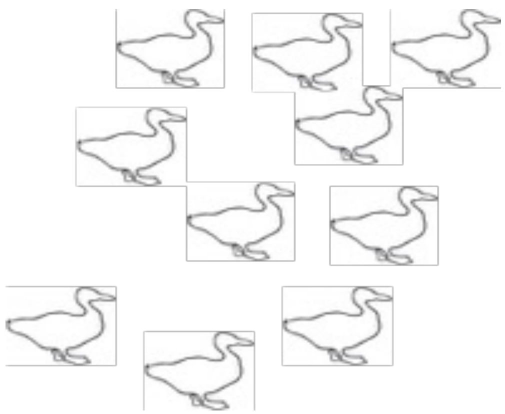

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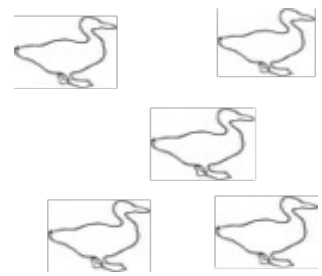

If you test your 10 ducks many times, on average you get 5

rbinom(200,10,0.5)

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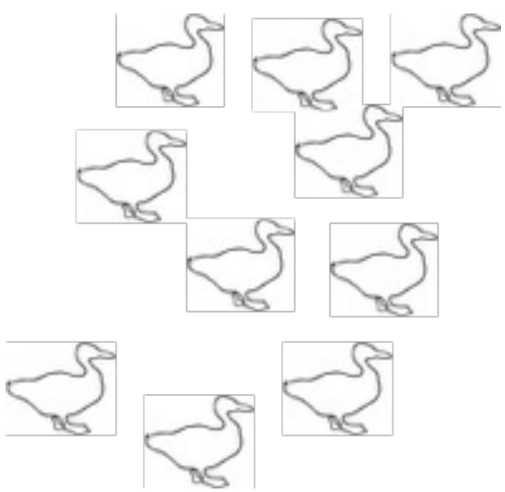

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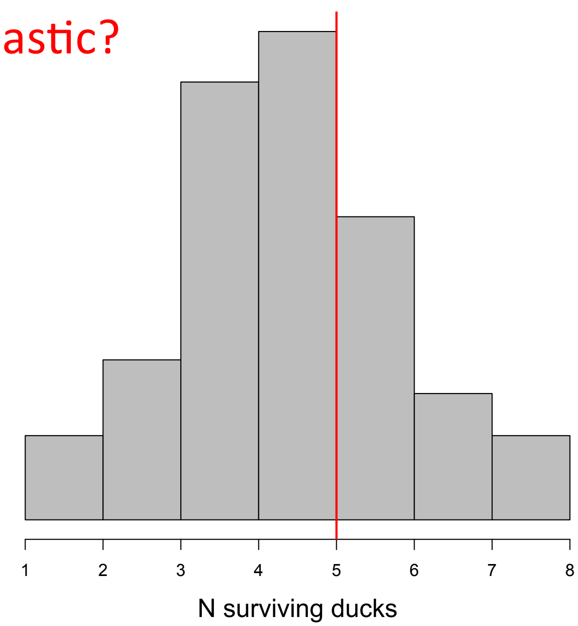
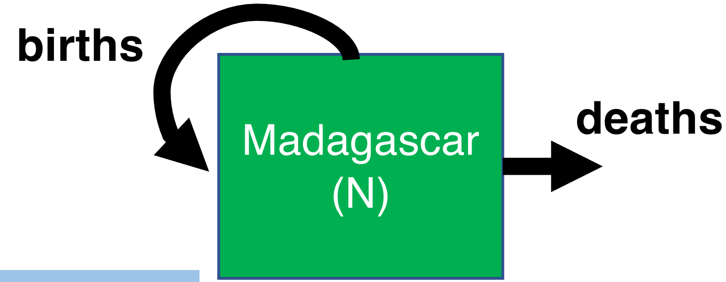


The basic population model

How are we doing?

Good! 🙂 ✅ ❤️ 👍

Bad! 👎 😮 ❌ 🤯
(or ask a question in the chat)



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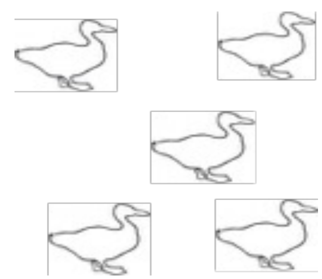

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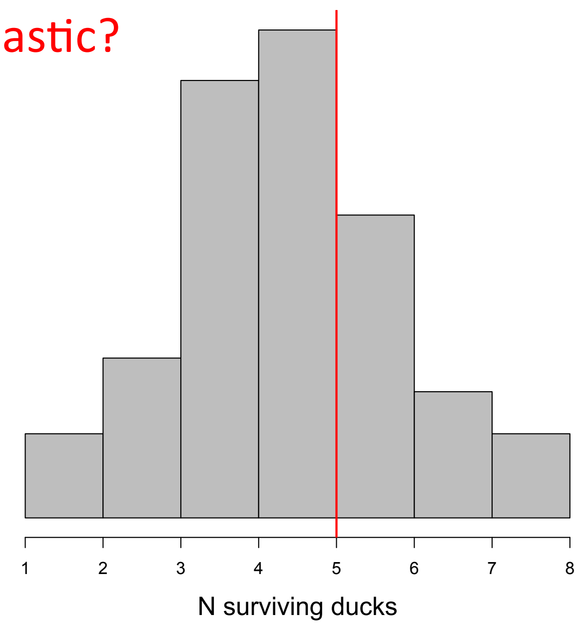

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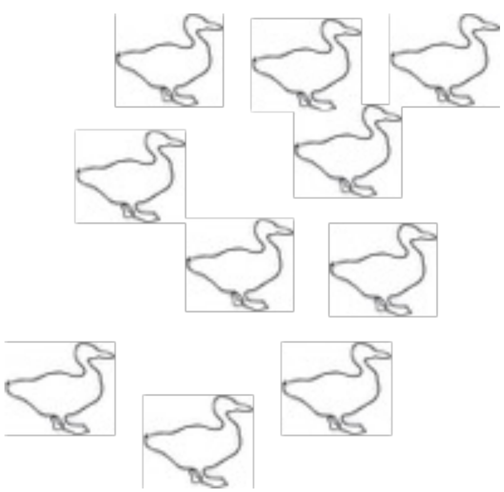

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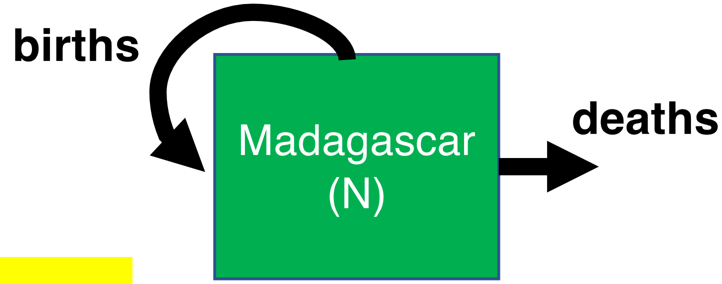
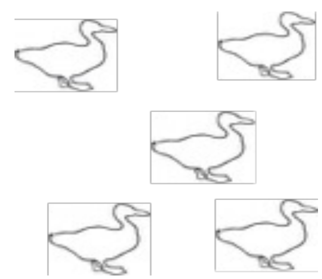
The basic population model



Question:

Does stochasticity matter more when the starting population is large or small?

if deterministic "always the same"



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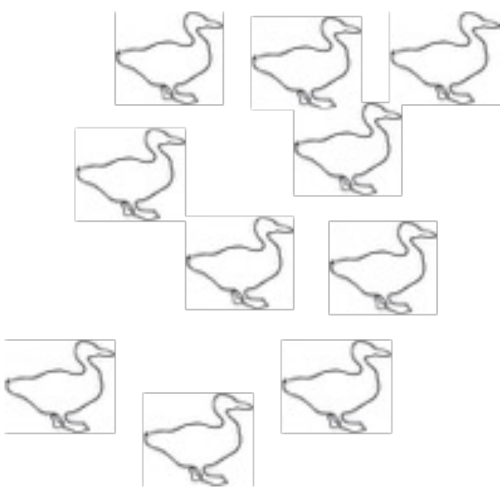

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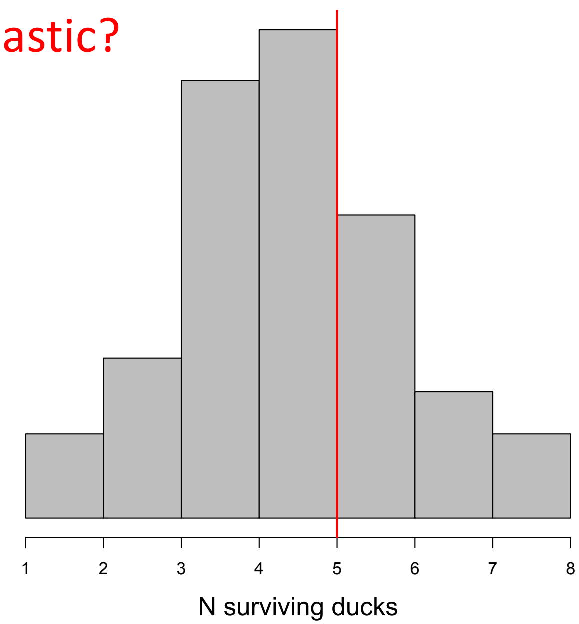


Key concepts

- Compartmental/mechanistic/mathematical models

    Modèles en compartiments

- Continuous vs. discrete models

    Modèles en temps continue vs. modèles en temps discrète

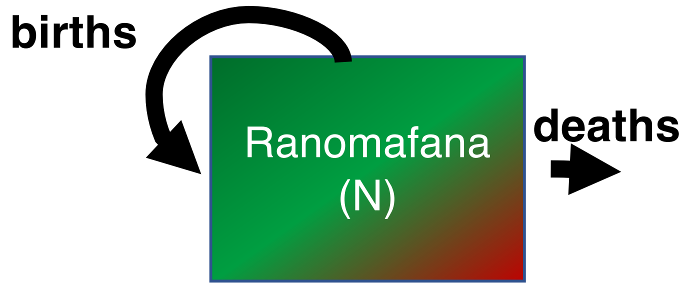
- Deterministic vs. stochastic models

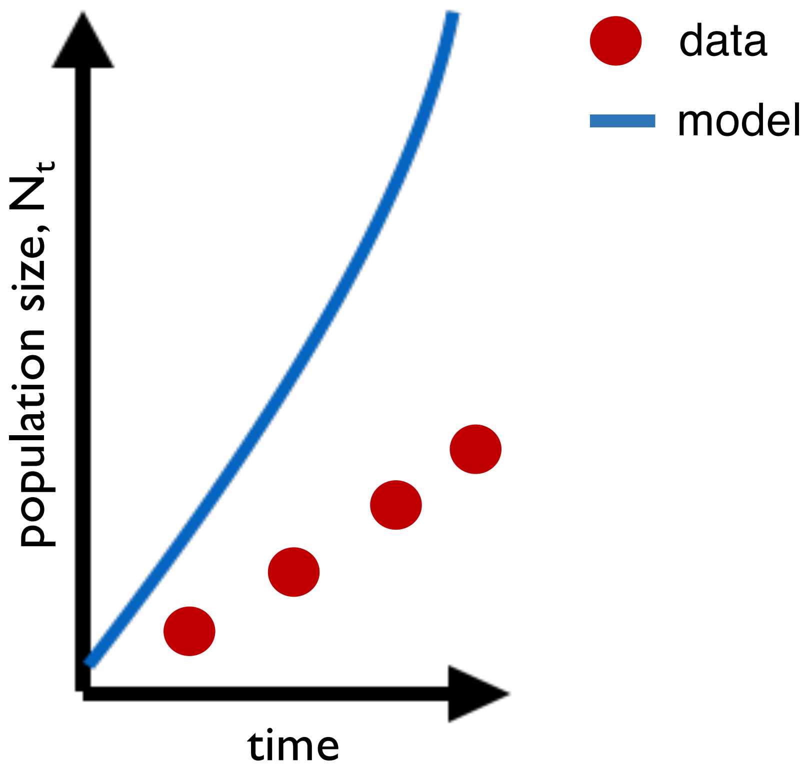
    Modèles détérministique vs. stochastique

2. Structured Population Models
2. Modèles de la population structurée

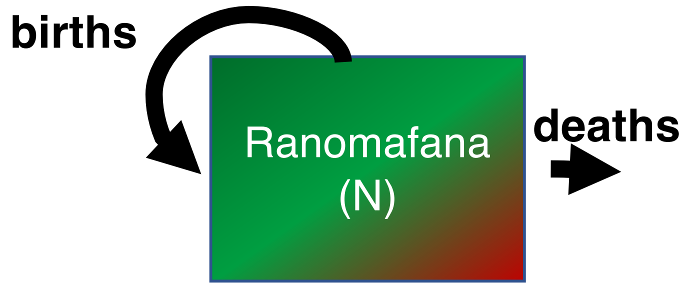
The structured population model

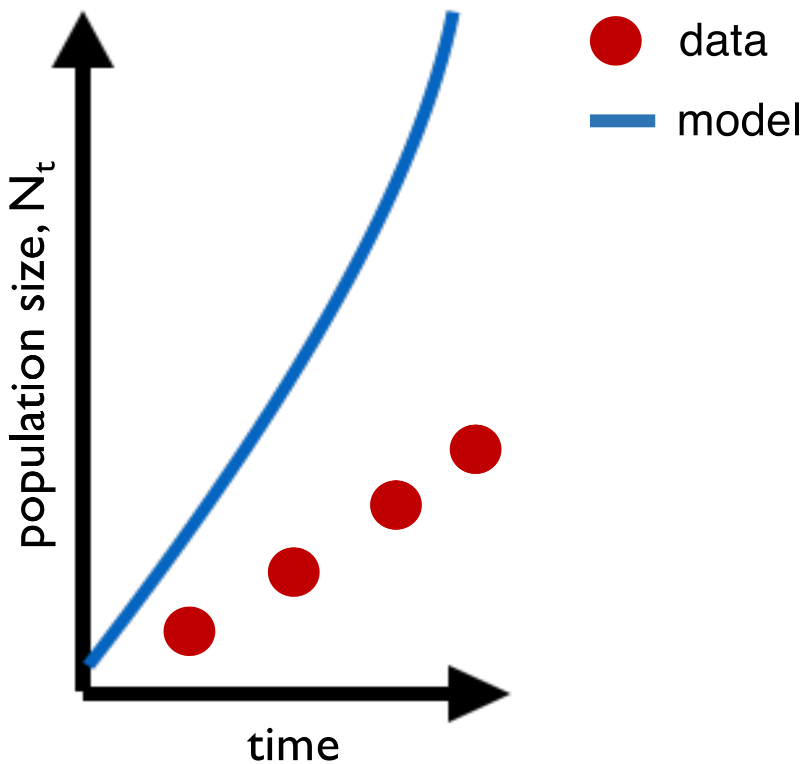
Why does the model perform poorly?





The basic population model





Why does the model perform poorly?

We need population structure!

The structured population model

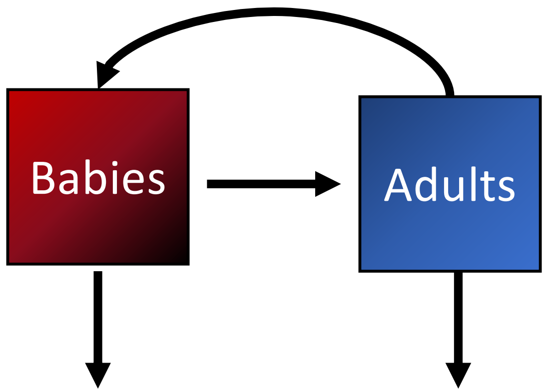
Compartmental models (Mechanistic Models)

1. Populations are divided into compartments
2. Individuals within a compartment are homogenously mixed
3. Compartments and transition rates are determined by biological systems
4. Rates of transferring between compartments are expressed mathematically

How does the population of Ranomafana grow over time?

Comment est-ce que la population de Ranomafana s'augmente avec le passage du temps?

The structured population model



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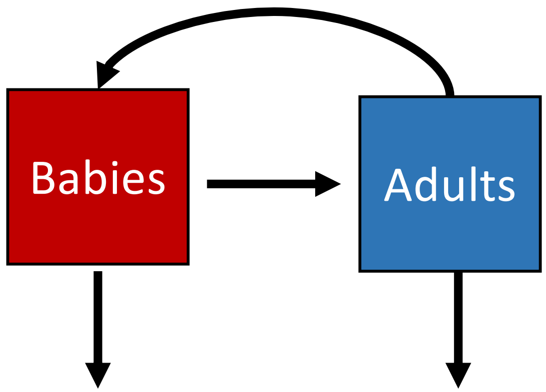

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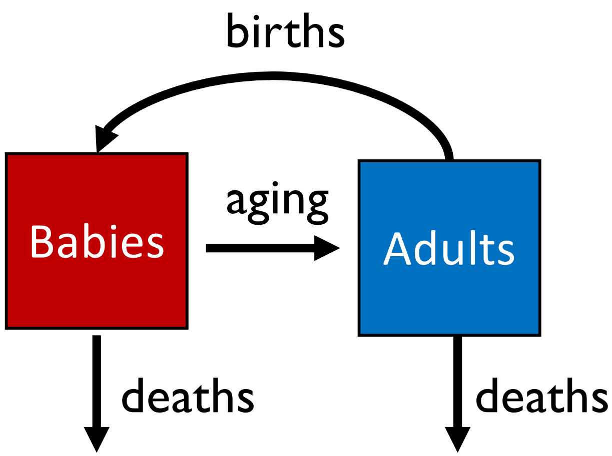

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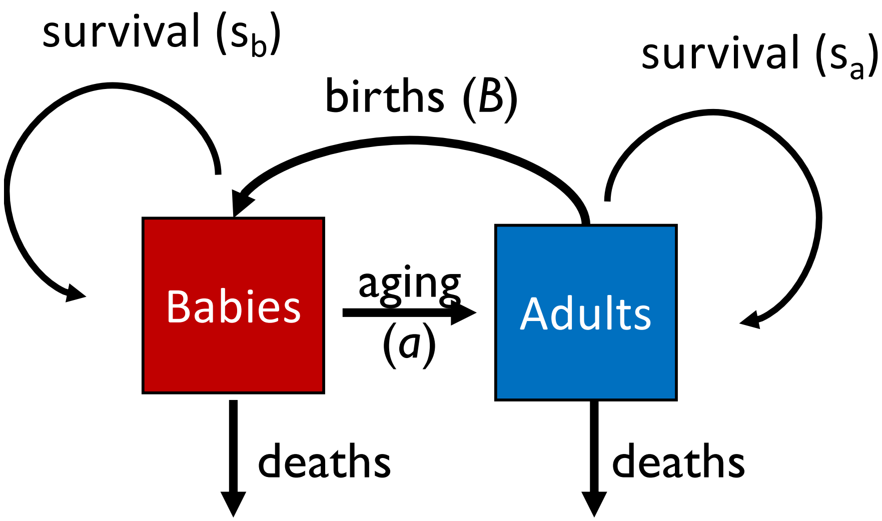

Population rate of increase

Taux d'accroissement de la population

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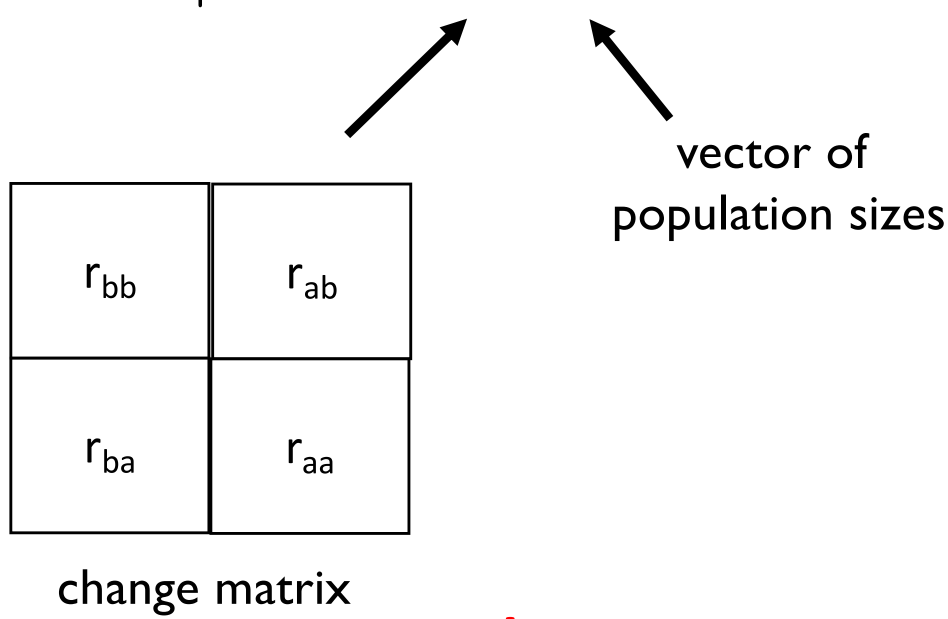
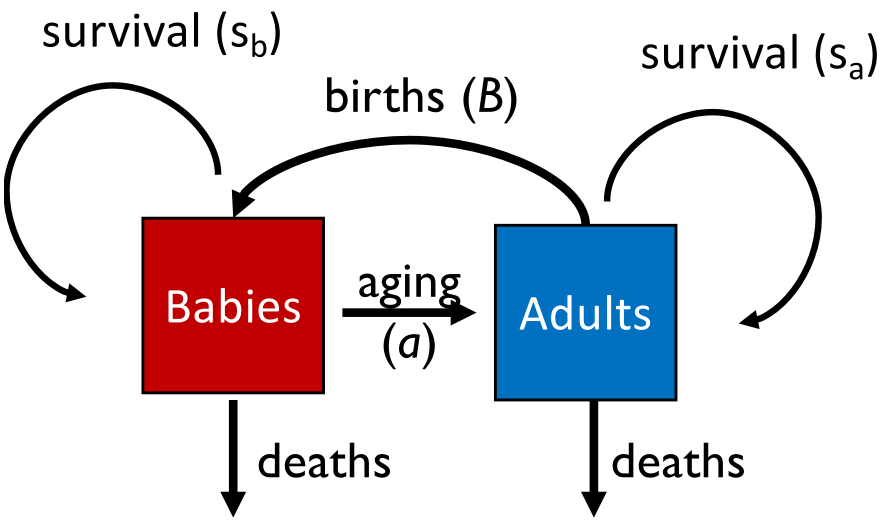
$$\mathbf{n_{t+1}} = \mathbf{A}\,\mathbf{n_t}$$



***Discrete time**

The structured population model



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$$\mathbf{n_{t+1}} = \mathbf{A}\,\mathbf{n_t}$$

vector of population sizes

$$\begin{bmatrix} r_{bb} & r_{ab} \\ r_{ba} & r_{aa} \end{bmatrix} = \begin{bmatrix} s_b(1-a) & B \\ s_b a & s_a \end{bmatrix}$$

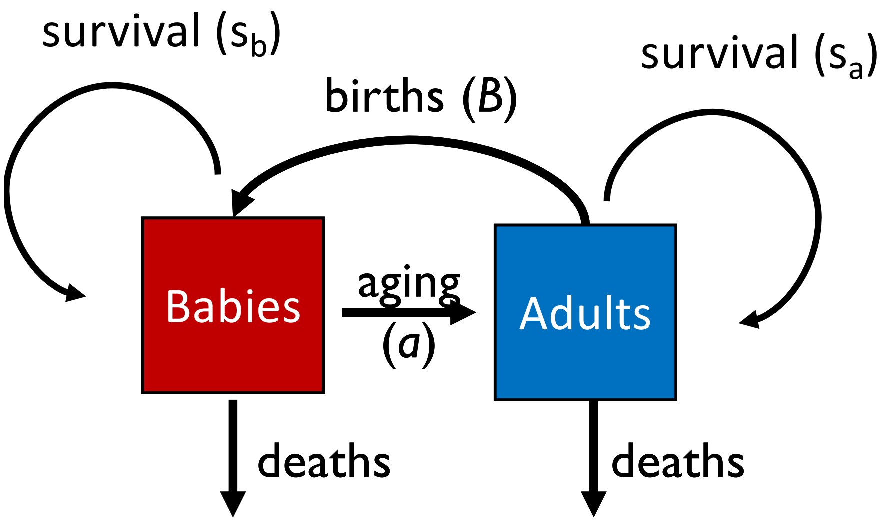
Population rate of increase

Taux d'accroissement de la population

change matrix

***Discrete time**

The structured population model



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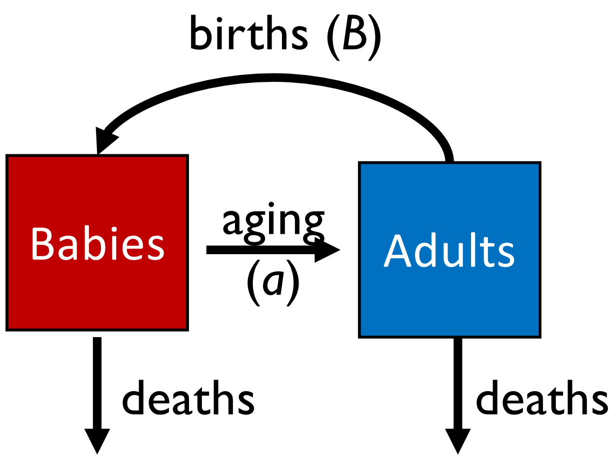
$$\mathbf{A n_t = n_{t+1}}$$

$$\underbrace{\begin{bmatrix} s_b(1-a) & B \\ s_b a & s_a \end{bmatrix}}_{\mathbf{A}} \times \underbrace{\begin{bmatrix} n_b \\ n_a \end{bmatrix}}_{\mathbf{n_t}} = \underbrace{\begin{bmatrix} s_b(1-a)\, n_b + b n_a \\ s_b\, a\, n_b + s_a n_a \end{bmatrix}}_{\mathbf{n_{t+1}}}$$

The structured population model



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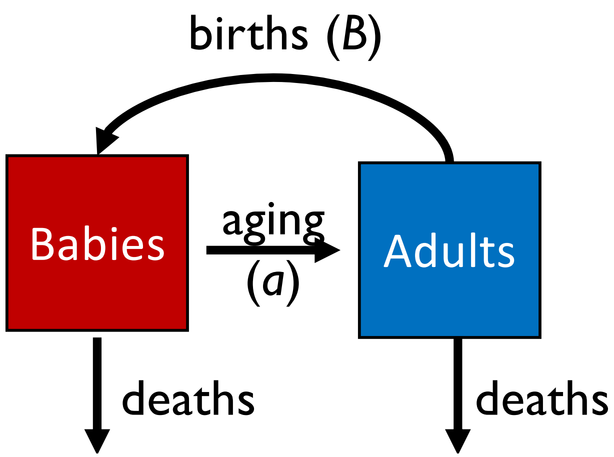
$$\mathbf{A}\mathbf{n}_t = \mathbf{n}_{t+1}$$

$$\underset{\mathbf{A}}{\begin{bmatrix} s_b(1-a) & B \\ s_b a & s_a \end{bmatrix}} \times \underset{\mathbf{n}_t}{\begin{bmatrix} n_b \\ n_a \end{bmatrix}} = \underset{\mathbf{n}_{t+1}}{\begin{bmatrix} s_b(1-a)\, n_b + b n_a \\ s_b\, a\, n_b + s_a n_a \end{bmatrix}}$$

Population growth will depend on population structure!

The structured population model



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Conservation and Management of a Threatened Madagascar Palm Species, *Neodypsis decaryi*, Jumelle

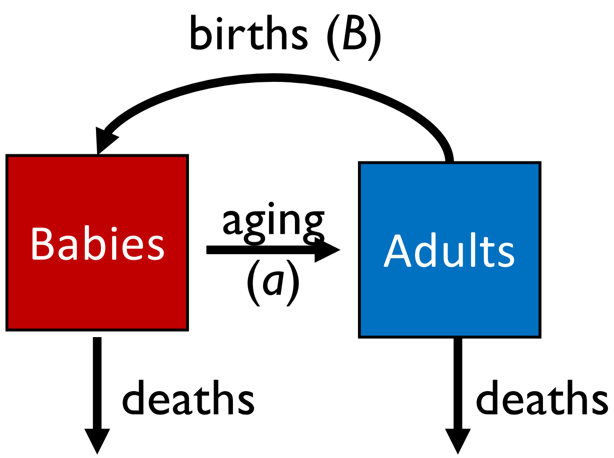
JOELISOA RATSIRARSON,*‡ JOHN A. SILANDER, JR.,* AND ALISON F. RICHARD†

*Department of Ecology and Evolutionary Biology, 75 N. Eagleville Road, The University of Connecticut, Storrs, CT 06269, U.S.A.

†Yale School of Forestry and Environmental Studies, 205 Prospect Street, New Haven, CT 06520, U.S.A.

‡Current Address: Yale School of Forestry and Environmental Studies, 205 Prospect Street, New Haven, CT 06520, U.S.A.

The structured population model



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How are we doing?

Good! 🙂 ✅ ❤️ 👍

Bad! 👎 😮 ❌ 🤯
(or ask a question in the chat)

Key concepts

- Compartmental/mechanistic/mathematical models

    Modèles en compartiments

- Continuous vs. discrete models

    Modèles en temps continue vs. modèles en temps discrète

- Deterministic vs. stochastic models

    Modèles détérministique vs. stochastique

- Structured models

    Modèles structurés.

R tutorial

Today

1. Solve a discrete time model two ways:
   1. Iteratively, using the change equation (for loop)
   2. Directly, using the state equation

2. Compare to the continuous time model

Wednesday

1. Solve a continuous time model three ways:
   1. Using the state equation
   2. Using a discrete-time approximation (for-loop)
   3. Using the ODE solver, lsoda

2. Use the ODE solver lsoda to simulate more complicated models:
   1. Predator-prey
   2. SIR